Ideal and Real Japanese Monetary Policy:  
A Comparative Analysis of Actual and Optimal Policy Measures  
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Abstract  
This paper discusses the successes and failures of Japanese monetary policy by evaluating policies from January 1980 to May 2003 in the light of optimal policy rules. First, we quantitatively conceptualize the Bank of Japan (BOJ)’s policy decisions by employing Bernanke and Mihov’s (1998) econometric methodology for developing monetary-policy measures, and term the resulting policy measure the ‘actual policy measure’. Next, assuming that the BOJ is committed to optimal policy rules, we simulate optimal policy paths, which we term ‘optimal policy measures’. We evaluate Japanese monetary policy historically by comparing actual and optimal policy measures.

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1 Introduction

Historical analyses of monetary policy generally require the use of particular policy rules as yardsticks. The comparison of an ideal policy path implied by a policy rule and an actual policy enables historical assessment of policy decisions and an exploration of the timing of policy mistakes. Taylor’s (1993, 1999) study of U.S. monetary history is an important and well-known example of comparative analysis. He introduced an interest-rate rule, known as the Taylor rule, which represents the response of a policy interest rate to a deviation of the inflation rate from its target value and to the output gap. Clarida et al. (1998, 2000) proposed a forward-looking Taylor rule to evaluate monetary policy quantitatively in industrialized countries. What the interest-rate rules have in common is that their adoption requires the presumption that policy rates, such as the federal funds rate and the call rate, represent central banks’ past policy decisions as prospective policy indicators. By considering the historically set values of the policy interest rate as actual policy decisions, while considering simulated values of the interest rate based on an interest-rate rule as optimal policy decisions, we can explore success and failure in the previous conduct of monetary policy. ¹

Recent studies of Japanese monetary policy have examined monetary history from the viewpoint of interest-rate rules. Bernanke and Gertler (1999) and Jinushi et al. (2000) used interest-rate rules based on the forward-looking Taylor rule. McCallum (2000, 2003) evaluated monetary history quantitatively from the viewpoint of the simple Taylor rule. ² These applica-

¹In general, monetary policy rules, such as the Taylor rule and the forward-looking Taylor rule, which directly express how policy rates as monetary-policy instruments change in response to target variables, have been referred to as instrument rules. Instrument rules do not necessarily require the use of policy rates as policy instruments. For example, McCallum (1988) proposed a monetary policy rule that requires the management of high-powered money rather than a policy rate.

²McCallum’s studies of Japanese monetary history adopted his own high-powered money rule as well as the Taylor rule. The high-powered money rule is referred to as
tions of interest-rate rules to Japanese monetary policy adopt the overnight call rate as the policy indicator of the Bank of Japan (BOJ). Furthermore, they assess Japanese monetary history by comparing the historically set values of the call rate with simulated values for the call rate based on interest-rate rules.

The use of inadequate policy indicators, which do not precisely reflect central banks’ past decisions, not only prevents a quantitative conceptualization of actual policy decisions, but also leads to erroneous assessments of past policy conduct. This means that accurate evaluation of monetary history requires an adequate policy indicator and the corresponding policy rule. The above studies of Japanese monetary history, which used interest-rate rules as yardsticks, depend on the assumption that the BOJ has always implemented policy changes by changing the call rate. That is, the behavior of the call rate reflects the BOJ’s policy decisions over time. However, in historical analyses of Japanese monetary policy, this assumption may not apply for three reasons. First, since July 1995, the call rate has hardly changed from around zero (see Figure 1). Second, since March 2001, the BOJ has adopted a new policy framework, which involves expanding the reserves as much as lowering the call rate. Third, previous studies of the BOJ’s policy indicators for the period up to June 1995, when the call rate remained positive and subject to change, do not necessarily support the view that only the call rate reflects the BOJ’s policy decisions. 3

These discussions suggest that the call rate alone may not be sufficient to explain the BOJ’s decisions over time. Therefore, the adoption of interest-rate rules involving the call rate would lead to incorrect assessments of the McCallum rule.

3Shioji (2000) and Nakashima (2005) explored policy indicators of the BOJ to June 1995 by using vector autoregression (VAR) methodology. Shioji concluded that the call rate and quantity indicators such as M2+CDs or high-powered money, may be useful as policy indicators of the BOJ. On the other hand, Nakashima concluded that the call rate is the best policy indicator of the BOJ.
ments of Japanese monetary policy. In this paper, we use the structural VAR methodology of Bernanke and Mihov (1998) to quantitatively conceptualize the BOJ’s policy decisions. Their methodology enables us to develop monetary policy measures of central banks by formulating equilibrium econometric models of the reserve market. In general, central banks aim to stabilize the macroeconomy by intervening in the reserve market and by setting reserves or short-term interest rates within a target range. Their methodology is convincing in that it assumes that monetary variables that are affected by the operating procedures of central banks in the reserve market embody the decisions of central banks.  

Bernanke and Mihov developed an equilibrium model of the U.S. reserve market to construct a quantitative policy measure of the Fed. In this paper, by thoroughly applying Bernanke and Mihov’s econometric methodology to Japanese monetary policy, we consider the following: (1) the institutional differences between the U.S. and Japanese reserve markets; and (2) the shift in the BOJ’s discount-window policy in July 1995. The former involves formulating a model of the Japanese reserve market that differs from that developed by Bernanke and Mihov for the U.S. reserve market. The latter involves modeling two equilibrium models of the Japanese reserve market: one each for before and after July 1995. In this paper, to identify the BOJ’s policy indicator, we present two original models of the Japanese reserve market. Each model captures the institutional features of the Japanese reserve market and reflects the difference in the BOJ’s operating procedures before and after the BOJ changed its discount-window policy. 

Bernanke and Mihov’s econometric methodology therefore does not assume the broad money supply such as M1 or M2 as policy indicators. Miyao (2005) claimed that Japan’s money supply has not worked not only as an intermediate target since the late 1970s (see Okina, 1993), but also as an information variable to predict future economic activity since the late 1990s. 

Nakashima (2005) analyzed the operating procedures of the BOJ up to June 1995 by employing Bernanke and Mihov’s econometric methodology. He excluded the period from
equilibrium models, we empirically demonstrate that individual monetary variables alone cannot explain the BOJ’s past policy decisions. Specifically, in the paper, we show that, in the subperiod to February 2001, the call rate alone could be used as the policy indicator of the BOJ. However, in the subperiod from March 2001, an equally weighted average of the call rate and reserves should be used. Furthermore, by applying the Bernanke–Mihov methodology, we identify the BOJ’s policy indicator for the entire period from January 1980 to May 2003. This indicator, termed the ‘actual policy measure’, is assumed to explain the BOJ’s historical decisions, or ‘real’ Japanese monetary policy.

In the context of a historical evaluation of Japanese monetary policy, how should we define optimal policy paths, or ‘ideal’ Japanese monetary policy, using the obtained actual policy measure? One possibility is to apply interest-rate rules. However, the application of an interest-rate rule is problematic. Since the components of the actual policy measure vary between periods, it is difficult to apply this rule. Furthermore, as Svensson (1997) has pointed out, coefficients of target variables in a policy response function characterized as an interest-rate rule are combinations of deep parameters, which define central banks’ weight preferences on the target variables or macroeconomic structures. This implies that we cannot use a central bank’s weight preference to simulate an optimal policy path.

In this paper, we adopt the so-called targeting rule, which requires the establishment of a central bank’s objective function with weight preferences on target variables. Specifically, we set up the BOJ’s objective function in the form of a quadratic loss function, which comprises deviations of the output gap and the rate of inflation from their target values. Moreover,

July 1995, and thus did not consider the shift in the BOJ’s discount-window policy in July 1995. In this paper, by including the period from July 1995, we consider the policy shift to develop a quantitative policy measure over time.
under the assumption that the BOJ commits itself to particular targeting rules, we simulate optimal policy paths, termed ‘optimal policy measures’, which represent past optimal policy decisions of the BOJ. Of course, decisions that are optimal at the time probably depend on the type of targeting rule to which the BOJ commits itself. Hence, we focus on three targeting rules: the ‘flexible targeting rule’, the ‘strict inflation-targeting rule’ and the ‘strict output-targeting rule’, each of which is characterized by the BOJ’s weight preferences on output and inflation stabilization. To evaluate Japanese monetary history, we compare actual and optimal policy measures, each obtained by adopting the three targeting rules as yardsticks. Specifically, this comparison shows that the historical evaluation of Japanese monetary policy, particularly from the early 1990s, depends on the targeting rules used as yardsticks. However, whatever targeting rules are adopted, we conclude that a tightening of monetary policy by the BOJ was delayed in the late 1980s, and that the BOJ’s policy stance in the early 1990s was too contractionary.

This paper is organized as follows. In Section 2, we determine the actual policy measure by applying the econometric methodology of Bernanke and Mihov. In Section 3, we determine optimal policy measures. In this section, we also examine success and failure in the past conduct of Japanese monetary policy by comparing actual and optimal policy measures. Section 4 concludes the paper.

2 Determining an Actual Policy Measure

Applying Bernanke and Mihov’s structural VAR methodology involves developing equilibrium econometric models of the reserve market. In particular, when modeling the Japanese reserve market, it should be noted that the introduction of a low interest-rate policy in July 1995 shifted the BOJ’s
discount-window policy. In this section, we determine the actual policy measure of the BOJ by considering the shift in the discount-window policy and the institutional differences between operating procedures in Japan and the U.S.

2.1 Bernanke and Mihov’s Methodology

To determine the actual policy measure of the BOJ, we follow Bernanke and Mihov in supposing that the economy is described by the linear structural model given by equations (1) and (2):

\[
Y_t = \sum_{i=0}^{k} B_i Y_{t-i} + \sum_{i=1}^{k} C_i P_{t-i} + A^v v_t^v
\] (1)

\[
P_t = \sum_{i=0}^{k} D_i Y_{t-i} + \sum_{i=0}^{k} G_i P_{t-i} + A^p v_t^p
\] (2)

where variables in bold type denote vectors or matrices.

Following Bernanke and Mihov, we refer to \(Y\) and \(P\) as ‘nonpolicy’ and ‘policy’ variables, respectively. The set of policy variables includes variables that are potentially useful as direct indicators of the stance of monetary policy, such as short-term interest rates and reserve measures. Nonpolicy variables include other economic variables, such as output and inflation. In equations (1) and (2), the \(v\)’s are mutually uncorrelated ‘structural’ or ‘primitive’ disturbances. In particular, one element of \(v_t^p\) is a money-supply shock or monetary-policy shock. The other elements of \(v_t^p\) may include shocks to money demand or any disturbance that affects the policy variables.

Bernanke and Mihov assumed that the nonpolicy variables, \(Y\), depend only on lagged values of the policy variables (\(C_0 = 0\)). Given the timing assumption, the system given by (1) and (2) can be rewritten in VAR form (with only lagged variables on the right-hand side) and estimated by standard methods. As in Bernanke and Mihov, let \(u_t^p\) be the parts of the VAR
residuals in the policy block that are orthogonal to the VAR residuals in the nonpolicy block. Then Bernanke and Mihov showed that $u_p^t$ satisfies:

$$(I - G_0)u_p^t = A^p v_p^t. \quad (3)$$

Equation (3) is a standard structural VAR system, which relates observable VAR-based innovations, $u$, to unobservable structural shocks, $v$. The Bernanke-Mihov methodology involves identifying exogenous components of monetary policy and examining policy indicators by developing equilibrium models of the reserve market in the form of (3). To model the Japanese reserve market, we discuss the BOJ’s operating procedures in the light of its discount-window policy.

### 2.2 The BOJ’s Discount-Window Policy

To develop an equilibrium model of the reserve market, we must understand the central bank’s policy behavior in the reserve market; i.e., how it supplies high-powered money (reserves plus currency).

In general, central banks have two ways of controlling the supply of high-powered money. One is to engage in open-market operations and the other is to engage in discount-window lending. In particular, management of the discount window takes two forms depending on the relationship between the discount rate and short-term policy rates such as the call rate and the federal funds rate. One form relates to the way in which a central bank sets the discount rate below the short-term policy rate. The other relates to the way in which it sets the short-term rate below the discount rate.

Historically, the BOJ has adopted both forms of discount-window policy. Figure 1 shows paths of the call rate and the discount rate, which indicate that the discount rate remained below the call rate until June 1995 and has remained above it since July 1995, when the BOJ implemented a low interest-rate policy. This suggests that management of the BOJ’s discount-
window policy before June 1995 was similar to that of the Federal Reserve (Fed), because the U.S. discount rate is persistently below the federal funds rate. However, there is an important difference. The BOJ eased (tightened) policy by increasing (reducing) discount-window borrowing quotas for private banks. Therefore, the BOJ took the initiative to control the level of discount-window lending and regulated the quantity of borrowing. Since, in Japan, moral suasion is not used in the manner used by the Fed to reduce discount-window borrowing, private banks usually borrow their quota amounts. In the literature on Japan’s monetary policy, this type of management of the BOJ’s discount window is generally termed ‘credit rationing’. 6 On the other hand, in the literature on U.S. monetary policy, it is supposed that borrowing from the Fed depends on private banks’ decisions, and that the Fed endogenously accommodates the demand for discount-window borrowing by private banks. To model the Japanese reserve market, we must consider differences between discount-window management in Japan and the U.S. 7

In July 1995, the BOJ, by setting the discount rate above the call rate, converted the discount rate into a penalty rate. The penalty rate eliminates the need for rationing at the discount window, and private banks usually have no incentive to borrow from the BOJ. Therefore, the BOJ’s discount window accommodates demand shocks for discount-window borrowing provided systemic risk in the short-term money market makes it difficult for private banks to obtain finance in this market.

Given the history of the BOJ’s discount-window policy, we must develop

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6Hamada and Iwata (1980) and Honda (1984) each developed theoretical models of the credit-rationing view, and the former used empirical analysis to support their view. Furthermore, Ueda (1993) stated “The discount rate has always been lower than the call rate. Therefore, discount-window lending has been rationed in Japan. And the level of lending has been changed by the BOJ, not by private banks” (p.12, lines 17–19).

7The type of discount-window policy pursued by the U.S. is generally referred to as the ‘implicit cost regime’ in the literature on Japan’s monetary policy.
two equilibrium models of the Japanese reserve market: for before and after June 1995. In the following subsections, we present two equilibrium models of the Japanese reserve market. One is the Credit Rationing (CR) model, applicable up to June 1995, and the other is the Low Interest-Rate (LIP) model, applicable from July 1995.

2.3 Before June 1995 – CR (Credit Rationing) Model

The following system, (4)–(9), describes the CR model:

\[
\begin{align*}
    u^{re} &= u^{br} + u^{mo} - u^{gd} - u^{cu} \\ 
    u^{gd} &= v^{gd} \\ 
    u^{cu} &= -\alpha u^r + v^{cu} \\ 
    u^{re} &= -\beta u^r + v^{re} \\ 
    u^{br} &= \phi^{gd} v^{gd} + \phi^{cu} v^{cu} + \phi^{re} v^{re} + \phi^{mo} v^{mo} + v^{br} \\ 
    u^{mo} &= \theta^{gd} v^{gd} + \theta^{cu} v^{cu} + \theta^{re} v^{re} + \theta^{br} v^{br} + v^{mo}
\end{align*}
\]

where \( gd, cu, re, br \) and \( mo \) denote government deposits, currency, reserves, borrowed reserves and assets held through open-market operations by the BOJ, respectively, and \( r \) denotes the call rate.

Equation (4) is the market-equilibrium condition for bank reserves, which is based on an identity between assets and liabilities on the BOJ’s balance sheet (see Table 1).\(^8\) Equation (5) implies that the BOJ accommodates fluctuations in the demand for government funds, \( v^{gd} \). Equation (6) relates innovations in the demand for currency, \( u^{cu} \), to innovations in the call rate, \( u^{re} \). Equation (7) is the market-equilibrium condition for borrowed reserves, which is based on an identity between assets and liabilities on the BOJ’s balance sheet (see Table 1).\(^8\) Equation (8) relates innovations in the demand for government funds, \( v^{gd} \), to innovations in the call rate, \( u^{re} \). Equation (9) relates innovation in the demand for currency, \( u^{cu} \), to innovations in the call rate, \( u^{re} \). It is important to note that, unlike the Fed, the BOJ has not used the concept of nonborrowed reserves. This is a major difference between the operating procedures of the BOJ and those of the Fed. Bernanke and Mihov’s econometric model of the U.S. reserve market incorporates an equilibrium condition for total reserves, (member-bank deposits plus vault cash) = borrowed reserves + nonborrowed reserves. Kasa and Popper (1997) have already applied the Bernanke–Mihov methodology to Japanese monetary policy. However, their analysis is deficient because it uses the concept of nonborrowed reserves: it does not take account of the institutional differences between Japan and the U.S.
and an autonomous shock to currency demand, $v^{cm}$. Similarly, equation (7) represents the bank’s demand for reserves, expressed in the form of innovations: it states that innovations in the demand for reserves, $u^{re}$, depend negatively on innovations in the call rate, $u^{r}$, and on a reserve demand shock, $v^{re}$.

Equation (8) represents the distinguishing feature of the CR model. It shows that the BOJ controls the level of discount-window lending and rationing lending to private banks. Hence, we interpret this equation as a behavior function for the BOJ. In particular, $v^{br}$ represents the supply shock for discount-window lending and is defined as a policy shock. Equation (9) is the second behavior function in the CR model, and shows how the BOJ supplies high-powered money by using open-market operations. In particular, $v^{mo}$ represents the high-powered money supply shock from using open-market operations and can be considered as the second monetary-policy shock, with $v^{br}$ in equation (8) being the first.

The CR model implies that the BOJ affects the short-term money market and the macroeconomy through both open-market operations and discount-window lending, because the model has two BOJ behavior functions. Furthermore, the two BOJ behavior functions are essentially equivalent in that they are high-powered money supply functions of the BOJ. Therefore, in the CR model, it is the quantity, rather than the composition, of high-powered money that matters. Hence, adding equations (8) and (9) yields the follow-

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Equation (8) indicates another major difference between the operating procedures of the BOJ and those of the Fed. In the literature on U.S. monetary policy, it is supposed that private banks are reluctant to borrow from the discount window because of various sanctions and restrictions imposed by the Fed on banks’ use of the window. Hence, the Fed only accommodates demand for discount-window borrowing by private banks. Specifically, Bernanke and Mihov used a conventional borrowing function, in which borrowing depends positively on the spread between the funds rate and the discount rate. Using data to June 1995, Nakashima (2005) found strong evidence against a model of the Japanese reserve market that incorporates a U.S.-type borrowing function rather than one of the form of equation (8).

11
ing system, which is essentially equivalent to the CR model:

\[
\begin{align*}
    u^{re} &= u^{md} - u^{gd} - u^{cu} \\
    u^{gd} &= v^{gd} \\
    u^{cu} &= -\alpha u^r + v^{cu} \\
    u^{re} &= -\beta u^r + v^{re} \\
    u^{md} &= \psi^{gd}v^{gd} + \psi^{cu}v^{cr} + \psi^{re}v^{re} + v^{md}
\end{align*}
\]

(10)

In this context, the VAR innovation, \( u^{md} \), is defined as follows:

\[
u^{md} = u^{br} + u^{mo}\]

The above system can be represented in the form of equation (3) as follows:

\[
I - G_0 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & \alpha & 0 \\
-1 & -1 & \beta & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\psi^{gd} & \psi^{cu} & \psi^{re} & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 \\
0 \\
-1 \\
0
\end{bmatrix}
= \begin{bmatrix}
\psi^{gd}v^{gd} + \psi^{cu}v^{cr} + \psi^{re}v^{re} + v^{md}
\end{bmatrix}
\]

Inverting the above relationship reveals how the monetary policy shock, \( v^{md} \), depends on the VAR innovations:

\[
v^{md} = - (\alpha \psi^{cu} + \beta \psi^{re})u^r + (1 - \psi^{re})u^{re} + (1 - \psi^{cu})u^{cu} + (1 - \psi^{gd})u^{gd}
\]

(11)

The CR model described by the above system has nine unknown parameters (including the variances of four structural shocks) to be estimated from 10 covariances. Hence, there is one overidentifying restriction.

2.4 After July 1995 – LIP (Low Interest-Rate Policy) Model

The following system of equations describes the LIP model:

\[
\begin{align*}
    u^{re} &= u^{br} + u^{mo} - u^{gd} - u^{cu}
\end{align*}
\]
The structure of the LIP model differs from that of the CR model in equation (12). This equation indicates that the BOJ passively accommodates the demand shock for discount-window borrowing by private banks, \(v^{br}\). Equation (13) represents the open-market operations behavior of the BOJ. The LIP model assumes that the BOJ can use only open-market operations to proactively supply high-powered money. Therefore, the high-powered money supply shock, \(v^s\), is defined as the monetary-policy shock of the BOJ in this model. Consequently, the LIP model can be written in the form of equation (3) as follows:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \alpha & 0 \\
0 & 0 & 1 & \beta & 0 \\
1 & 1 & 1 & -\gamma & -1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\theta^{gd} & \theta^{cu} & \theta^{re} & \theta^{br} & 1
\end{bmatrix}
\]

One can also invert the above relationship to determine how the monetary policy shock, \(v^s\), depends on the VAR innovations:

\[
v^s = -(\alpha \theta^{cu} + \beta \theta^{re})u^r + (\theta^{br} + 1)v^{mo} - (\theta^{br} + \theta^{re})u^{re} - (\theta^{br} + \theta^{cu})u^{cu} - (\theta^{br} + \theta^{gd})u^{gd}
\]

The LIP model described by the above structural VAR system has 11 unknown parameters (including the variances of five structural shocks) to be estimated from 15 covariances. Hence, there are four overidentifying restrictions.
2.5 Theoretical Models for Alternative Operating Procedures

Parameters in the BOJ behavior functions, given by equation (10) in the CR model and by equation (13) in the LIP model, define how the BOJ controls the market for bank reserves in each model. For example, the proposition that the BOJ targets only the call rate can be represented by three additional restrictions in the CR model, $\psi^{gd} = 1$, $\psi^{cu} = 1$ and $\psi^{re} = 1$, and by four additional restrictions in the LIP model, $\theta^{gd} = 1$, $\theta^{cu} = 1$, $\theta^{re} = 1$ and $\theta^{br} = -1$. In this case, the monetary policy shocks can be recovered by using the VAR innovations to the call rate. According to this proposition, the call rate provides the best policy indicator of the BOJ.

Alternative propositions that define the BOJ’s policy in terms of both the call rate and quantity indicators, such as currency and reserves, can also be represented by parametric restrictions in the BOJ behavior functions. For example, the proposition that the BOJ targets both the call rate and reserves can be written in terms of two additional restrictions in the CR model, $\psi^{gd} = 1$ and $\psi^{cu} = 1$, and three additional restrictions in the LIP model, $\theta^{gd} = 1$, $\theta^{cu} = 1$ and $\theta^{br} = -1$. In this case, the policy shocks can be recovered by using linear combinations of the VAR innovations to the call rate and reserves. According to this proposition, a hybrid variable comprising the call rate and reserves provides a good policy indicator of the BOJ. Hence, imposing various parametric restrictions on equations (10) and (13), respectively, yields six alternative models that are nested within the CR and LIP models. In particular, we describe two of the six models as single-targeting models, which assume that the BOJ targets a single monetary variable, while four are described as mixed-targeting models, which assume that the BOJ targets a combination of the following policy-sector variables: the call rate, currency, reserves and high-powered money (currency plus reserves). Table 2 presents the models, which imply different
forms of BOJ operating procedures. In what follows, we examine the BOJ’s policy indicator by estimating the six alternative models and the CR and LIP models.

2.6 Data, Estimation and Results

As explained in Subsection 2.1, the Bernanke–Mihov methodology accommodates the inclusion of both policy variables and nonpolicy variables in the VAR system. Included in the nonpolicy sector are the output gap \( y \) for the industrial production index (1995=100, seasonally adjusted) and the rate of inflation \( \pi \) in the consumer price index (1995=100, excluding food products). The consumer price index is seasonally adjusted by the X11 method. The output gap was measured by using percentage deviations from the trend, which was constructed by using the Hodrick–Prescott filter. \(^{10}\) The inflation rate is annual.

Consider the policy variables in the VAR system. As discussed in Subsections 2.3 and 2.4, the development of equilibrium models of the market for bank reserves involves the use of identities between assets and liabilities in the BOJ’s balance sheet (Table 1). Government deposits (GD), currency (CU) and reserves (RE) are used for liabilities. Furthermore, ‘the assets held via open-market operations (MO)’, which comprise bills, bonds and overseas assets acquired by the BOJ through these operations, are used for assets. \(^{11}\) In addition to these four variables, the call rate \( R \) is included in the policy sector. \(^{12}\) Therefore, we estimate the seven-variable VAR system for \( y, \pi, \)

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\(^{10}\) To apply the Hodrick–Prescott filter, we used a value of the smoothing parameter for monthly data of 129,600, as proposed by Ravin and Uhlig (2002).

\(^{11}\) Each equilibrium condition in the CR and LIP models requires one of the quantity variables in the policy sector to be redundant. Therefore, we exclude borrowed reserves (BR) from the policy sector.

\(^{12}\) For details of MO, see Appendix A. Normalizing the policy-sector variables, except for \( R \), causes log-linear estimation to violate the identity relationship between assets and liabilities. To deal with this problem, Bernanke and Mihov suggested that the policy-sector variables should be normalized by using a 36-month moving average of past values.
GD, CU, RE, R and MO. All data were obtained from the Nikkei NEEDS, and the sample period is from January 1976 to May 2003. To determine the number of lags in the VAR systems, we applied the Akaike Information Criterion (AIC). This criterion suggested 15 lags.

To estimate the CR and LIP models, we use a two-step procedure. In the first step, we estimate the reduced-form VAR by using equation-by-equation OLS estimation for the full-sample period from 1976 to 2003. OLS estimation generates five policy-sector VAR innovations: $u_{gd}$, $u_{cu}$, $u_{re}$, $u_r$ and $u_{mo}$. For post-1995 estimation of the LIP model, we can use the five VAR innovations. However, pre-1995 estimation of the CR model requires the construction of $u_{md}$, which is the VAR innovation of ‘the assets held via open-market operations and discount-window lending (MD)’. To obtain $u_{md}$, after generating $u_{br}$ from $u_{gd}$, $u_{cu}$, $u_{re}$ and $u_{mo}$ by using the market-equilibrium condition in equation (4), we add $u_{br}$ and $u_{mo}$. Therefore, for pre-June 1995 estimation of the CR model, we use the four policy-sector VAR innovations of $u_{gd}$, $u_{cu}$, $u_r$ and $u_{md}$. In addition, for post-1995 estimation of the LIP model, we must take it into account that, in March 2001, the BOJ officially adopted a new operating procedure by targeting the total reserves. We adopt this approach by normalizing the policy-sector variables by using a 36-month moving average of past values of ‘the BOJ’s assets held via open-market operations and discount-window lending (MD)’, generated by summing MO and discount-window lending. For details of MD, see Appendix A.

We can also estimate the CR model by directly using ‘the assets held via open-market operations and discount-window lending (MD)’, which comprises MO and discount-window lending. This requires estimation of the six-variable VAR system (in $y$, $\pi$, GD, CU, R and MD) for the CR model. The author confirms that there is no significant difference between the estimation results for the CR model based on the six-variable VAR and those based on the seven-variable VAR. However, the use of the six-variable VAR requires estimation of separate VAR systems for the pre- and post-1995 periods. Therefore, the CR and LIP models on different VAR systems not only differ in their contemporaneous structures of the reserve market, but also in their dynamic structures of macroeconomy. Given that we are attempting to identify a useful policy measure over time by focusing on the difference in the contemporaneous structure of the reserve market before and after 1995, it is important to minimize differences between the two models through the use of a single VAR system. This approach is adopted in this paper.
level of reserves as much as by continuing with the so-called zero interest-rate policy. We should carefully examine whether the LIP model can capture this change in the BOJ’s operating procedures in March 2001. Hence, for pre- and post-2001 estimation of the LIP model, we split the five policy-sector VAR innovations, which is generated for post-1995 estimation of the LIP model, at March 2001.

In the second step, full-information maximum likelihood estimation is applied to the structural VAR system of equation (3). The log likelihood function to be maximized is as follows:

\[
L(I - G, A, \Sigma_u) = -(T/2)\{ \log |I - G|^2 - \log |A|^2 - \log |\Sigma_u|^2 \} - (T/2)\text{trace}\{ (I - G)'(A^{-1})'\Sigma_u^{-1}A^{-1}(I - G)\Sigma_u \}
\]

where \( \Sigma_u \) is the estimate of the covariance matrix of the policy-sector VAR innovations and \( \Sigma_u \) is the diagonal matrix that diagonally locates the variances of the structural shocks. Following Bernanke and Mihov, we performed two types of test on the models: (1) tests of the validity of the full set of overidentifying restrictions; and (2) joint hypothesis tests on the structural parameters, conditional on the validity of both the CR and LIP models.

For ease of interpretation, we define ‘weighting parameters’, \( \omega \), in the CR, LIP and mixed-targeting models. The weighting parameters are the absolute values of the parameters corresponding to the VAR innovations in the BOJ behavior functions, (10) and (13). The absolute values in the behavior functions are normalized to sum to unity. For example, the weighting parameters in the CR model are:

\[
v^md = \omega^r u^r + \omega^{re} u^{re} + \omega^{cu} u^{cu} + \omega^{gd} u^{gd},
\]

where \( \omega^r + \omega^{re} + \omega^{cu} + \omega^{gd} = 1 \). Each \( \omega \) satisfies \( \omega^r = \left| (\alpha \psi^{cu} + \beta \psi^{re}) / k \right| \), \( \omega^{re} = \left| (1 - \psi^{re}) / k \right| \), \( \omega^{cu} = \left| (1 - \psi^{cu}) / k \right| \) and \( \omega^{gd} = \left| (1 - \psi^{gd}) / k \right| \), where
\[ k = |\alpha \psi^u + \beta \psi^r| + |1 - \psi^r| + |1 - \psi^c| + |1 - \psi^g|. \]  

14 Similarly, the weighting parameters in the LIP model are:

\[ v^s = \omega^r u^r + \omega^{mo} u^{mo} + \omega^e u^e + \omega^c u^c + \omega^g d^g, \]

where \( \omega^r + \omega^{mo} + \omega^e + \omega^c + \omega^g = 1 \). Each \( \omega \) satisfies \( \omega^r = \alpha \theta^r u^r + \beta \theta^r u^r + \theta^r + \theta^c \) and \( \omega^g = \theta^g + \theta^c \) and \( \omega^g = \theta^g + \theta^c + \theta^g + \theta^g \). The weighting parameters in each of the four mixed-targeting models, including the CL–CU–RE, CL–CU, CL–RE and CL–HP models, shown in Table 2, are defined similarly.

15 In all 14 models, the parameter estimates of the demand functions are of the expected sign, although some estimates are not statistically significant.

1. For the pre-June 1995 period, the parameter estimates of the BOJ behavior function in the CR and mixed-targeting models are close to unity. This is consistent with the CL model being the most easily accepted, and with the estimates of \( \omega^r \) being relatively high. Furthermore, all other models except the HP model are easily accepted. These results indicate that the call rate is the best policy indicator of the BOJ for the period before June 1995.

2. For the post-July 1995 period, estimation results for before and after March 2001 differ. For the first subperiod, before March 2001, the parameter estimates of the BOJ behavior function in the LIP and mixed-targeting models are close to unity. In addition, the CL model is easily accepted, whereas the HP model is strongly rejected. Taking the pre-June 1995 results into account, we conclude that the call rate represents the BOJ’s actual policy variable for the pre-February 2001 period, including the pre-June 1995 period.

3. For the second subperiod, from March 2001, the single-targeting models, the CL and HP models, are rejected at the five-percent level of significance. However, the LIP model and the two mixed-targeting
models, the CL–CU–RE and CL–RE models, are easily accepted. In particular, the estimates of \( \omega \) in the three models indicate that, since March 2001, the BOJ has been equally concerned about the call rate and reserves. These results suggest that an equally weighted average of the call rate and reserves can be used as the policy indicator of the BOJ in this period.  

2.7 The Actual Policy Measure of the BOJ

The estimation results suggest that the composition of the BOJ’s policy measure might differ between periods. Hence, to calculate a useful policy measure over time, we apply the method proposed by Bernanke and Mihov. First, we calculate the sum of the policy shock and the corresponding element of \( A^{-1}(I - G)P_t \). Specifically, in terms of equation (3), this is the fourth element in the context of the CR model and its six associated models, and is the fifth element in the context of the LIP and its six associated models. For the CR model, \( P_t \) includes the four policy variables, R, RE, CU and GD, while for the LIP model, \( P_t \) includes the five policy variables, R, MO, RE, CU, GD. This procedure generates seven series in the three periods, pre-June 1995, July 1995 to February 2001 and post-March 2001. Next, we normalize the \( p \)-values of the tests of the overidentifying restrictions performed in each subperiod so that they sum to unity. Using the normalized values, we derive a weighted average policy measure in each period. Then, we normalize the calculated policy measure at each date by subtracting it from a 36-month moving average of its own past values over the entire period. This implies that zero is the benchmark for ‘normal’ monetary policy.

\[\text{For robust checking of the results for the three subperiods, VAR systems are estimated with 12, 13, 14, 16, and 17 lags. The ordering of the VARs does not materially affect the results.}\]
(normal at least in terms of recent experience). The historical values are referred to as the ‘actual policy measure’ of the BOJ. Figure 2 shows the obtained policy measure from January 1980 to May 2003. Several features are noteworthy.

1. After a temporary tightening immediately following the Plaza Agreement of September 1985, the policy stance in the late 1980s was substantially expansionary.

2. In the early 1990s, when the bubble economy of the late 1980s burst, the policy stance was contractionary.

3. In the mid 1990s, the policy stance was expansionary at the beginning of the period of the low interest-rate policy in June 1995. After that, the policy stance in the late 1990s was neutral.

4. Except for a temporary tightening involving raising the call rate in August 2000, the policy stance has been expansionary since the beginning.

---

17 The issue of how we should define normal (or neutral) policy is an important macroeconomic issue. Following Bernanke and Mihov, we use the 36-month moving average method to derive a neutral policy indicator in each month. As an alternative to our neutral policy indicator, arguably, the natural rate of interest, which is the real short-term interest rate consistent with output being at its natural rate and inflation being constant, is the most appropriate indicator of neutral policy (see, e.g., Blinder (1998)). However, the use of the natural real interest rate is problematic for two reasons. First, the standard approach to calculating the path of the natural rate has not yet been established. As Laubach and Williams (2003) have pointed out, econometric estimates of the natural rate of interest are imprecise. Second, quantitative conceptualization of the BOJ’s policy stance based on the natural rate of interest is quite inconsistent with the framework of this paper. This is because the BOJ’s policy indicator obtained by using estimates from the CR, LIP and their associated models implies different monetary indicators, including the call rate, reserves and currency, which are not measured in comparable units. The moving average method proposed by Bernanke and Mihov has the advantage of alleviating this problem of inconsistent units of measurement. Hence, in what follows, we measure the BOJ’s policy stance based on the neutral policy indicator obtained by using the moving average method.

18 Positive values indicate an easing of monetary policy, while negative values indicate a tightening. The actual policy measure is scaled so that it has the same variance as the call rate.

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3 Actual and Optimal Policy Measures

This section simulates optimal policy decisions by using various types of targeting rule, which requires that we set up the BOJ’s objective function. In addition, in this section, we evaluate Japanese monetary history by comparing the resulting optimal policy paths, referred to as ‘optimal policy measures’, with the actual policy measure.

3.1 Calculating Optimal Policy Measures

Similarly to Bernanke and Mihov’s VAR system, given by equations (1) and (2), we assume that the economy is described by the following linear structural model:

\[
Y_t = \sum_{i=0}^{k} B_i Y_{t-i} + \sum_{i=1}^{k} C_i p_{t-i} + A^Y v^v_t
\]

\[
p_t = \sum_{i=0}^{k} D_i Y_{t-i} + \sum_{i=1}^{k} G_i p_{t-i} + v^p_t,
\]

where \(p_t\) is the actual policy measure. The vector of nonpolicy variables in the VAR is denoted by \(Y'_t = (y_t, \pi_t)\), which includes the output gap for industrial production (\(y\)) and the rate of inflation (\(\pi\)). The AIC suggests the use of 12 lags in the VAR system. In what follows, we calculate the BOJ’s optimal policy measures, taking as given the estimated dynamic behavior of the nonpolicy sector in the three-variable VAR system. This implies that the BOJ makes optimal policy decisions by monitoring the behavior of two nonpolicy variables, the output gap and inflation.

\[19\] Sack (2000), using a targeting rule, derived optimal paths for the federal funds rate. He evaluated U.S. monetary history, particularly in the post-Volcker period, by comparing historically set values of the funds rate with the values implied by the optimal funds rate paths.
The following objective function is assumed for the BOJ:

$$E_t \sum_{i=1}^{\infty} \beta^i [\lambda y_t^2 + (1 - \lambda)(\pi_t - \pi^*)^2]$$

where \( \beta \geq 0 \) is a discount factor. The parameter \( \lambda \geq 0 \) is the weight on output and inflation stabilization, and \( \pi^* \) is the BOJ’s target rate of inflation.

To solve the optimization problem, we define a state vector that includes current and lagged values of the output gap and inflation, and lagged values of the actual policy measure as follows:

$$X'_t = \{y_t, y_{t-1}, \ldots, y_{t-12}, \pi_t, \pi_{t-1}, \ldots, \pi_{t-12}, p_t, p_{t-1}, p_{t-2}, \ldots, p_{t-12}, 1\}.$$  

The optimal policy is a solution to the following Bellman equation:

$$V(X_t) = \min_{p_t} \{(X_t - X^*)'Q(X_t - X^*) + \beta E_t[V(X_{t+1})]\}$$

subject to

$$X_{t+1} = RX_t + SP_t + \epsilon_{t+1}. \quad (15)$$

where \( Q \) is a matrix that has relative weights corresponding to the output gap and inflation on the leading diagonal, and zeros elsewhere. The dynamics of the state vector are governed by the matrix \( R \) and the vector \( S \), which incorporate the point estimates of the coefficients from the nonpolicy sector in the VAR.

Since the per-period payout is quadratic and the dynamics are linear, the value function has the following form:

$$V(X) = X'AX + 2X'\Omega + \beta(1 - \beta)^{-1}\text{trace}(AXS) \quad (16)$$

where \( \Sigma = E(\epsilon_t \epsilon'_t) \) is the covariance matrix of the disturbance vector.

It can be shown that the solution to this problem is given by:

$$p_t^{optimal} = -(S'\Sigma S)^{-1}[(S'\Lambda R)X_t + S'\Omega] \quad (17)$$
where $p_{t}^{optimal}$ denotes an ‘optimal policy measure'. The matrix $\Lambda$ must satisfy the following Riccati equation:

$$\Lambda = -Q + R'AR - R'AS(S'AS)^{-1}S'AR.$$  

In addition, $\Omega$ satisfies the following:

$$\Omega = \{I - R'[I - \Lambda S(S'AS)^{-1}S']\}^{-1}QX^*.$$  

In equation (17), the optimal policy measure is a function of all current and lagged values of the nonpolicy variables and lagged values of the actual policy measure. The behavior of the optimal policy measure depends on the three parameters in the problem, $\beta$, $\lambda$ and $\pi^*$. Following Rudebusch and Svensson (1999), a discount factor of $\beta = 1$ is henceforth imposed.  

The remaining parameters are choice variables of the BOJ. For the target rate of inflation, we assume $\pi^* = 1.64$, which is the sample average of $\pi$ for the period from January 1980 to May 2003.  

In the following subsection, we calculate the optimal policy measure for different values of $\lambda$. Furthermore, we compare the calculated optimal policy measures with the actual policy measure.  

### 3.2 Comparative Analysis of Actual and Optimal Measures

For an optimal policy measure in the form of (17), the dynamics from model (15) are as follows:

$$X_{t+1} = VX_t + W + \epsilon_{t+1} \quad (18)$$

---

20 The results are not sensitive to the value of the discount factor. Rudebusch and Svensson showed that for $\beta = 1$, the optimal value function (16) reduces to $V(X) = \text{trace}(\Lambda \Sigma)$. We have used this function to calculate the optimal values in Table 5.

21 Clarida et al. (1998) explained that, if the sample period is sufficiently long, their method produces a target rate of inflation, $\pi^*$, that is similar to the sample average. Indeed, by applying this method, Bernanke and Gertler (1999) obtained $\pi^* = 1.73$ for the period from April 1979 to December 1998. This estimated value is close to the sample average value of inflation calculated in this paper.

22 The way the optimal policy measures are computed is subject to the Lucas (1976) critique. See Appendix B for details.
where \( V = R - S(S'AS)^{-1}(S'\Lambda R) \) and \( W = -S(S'AS)^{-1}S'\Omega \). In equation (18), the first and second elements of \( X \) represent optimal paths for the output gap and inflation, respectively, under an optimal policy rule. In the following analysis, we compare the simulated paths for the output gap and inflation as well as for an optimal policy with their actual paths, for a given value of \( \lambda \). \(^{23}\)

First, we analyze the ‘flexible targeting rule’, in which \( \lambda = 0.5 \), and consider this case as a benchmark. The justification is that, in this case, the central bank cares about output stabilization as well as inflation stabilization, which is realistic. \(^{24}\) The bold lines in Figure 4 represent the optimal paths for policy, output and inflation, while the thick lines represent the actual paths. In particular, the optimal policy path represents the policy measure, \( p_t^{optimal} \), that is predicted by the flexible targeting rule given the state of economy in each month. In other words, the optimal policy path indicates that, for any particular month, the BOJ makes the optimal decision given that it has previously implemented its actual policy. The following features are noteworthy.

1. In the late 1980s, in the period of the bubble economy, monetary tightening was delayed.

2. In the early 1990s, when the bubble economy burst, the BOJ’s actual

\(^{23}\)As expected, the third element of \( X \) reduces the actual policy measure. Equation (18) is referred to as an optimal closed system. See Ljungqvist and Sargent (2000, Chap. 4) for details.

\(^{24}\)Figure 3 shows the trade-off between inflation variability and output gap variability for weights on output stabilization (\( \lambda \)) ranging from 0 to 1 in intervals of 0.001. Table 5 indicates the performance of various rules for several illustrative cases. The results are based on unconditional covariance matrices for \( X \) in equation (18), which are calculated by using the doubling algorithm described in Anderson et al. (1996). Figure 3 indicates that, as \( \lambda \) decreases, the optimal rule corresponds to points further southeast on the curve. In particular, points A, D and G correspond to strict output targeting, flexible targeting and strict inflation targeting, respectively. Each figure and table provides empirical justification for treating the flexible targeting rule as the benchmark.
policy stance might have been too contractionary.

3. From the early to the mid 1990s, an easing of monetary policy was delayed; i.e., the BOJ should have implemented a more expansionary policy.

4. From 1996 to 1998, the BOJ should have tightened policy to reduce output and inflation.

5. From the late 1990s, the BOJ should have implemented a more expansionary policy to alleviate the deflationary recession.

Next, we analyze two extreme cases, represented by $\lambda = 0.0$ and 1.0. Figure 5 illustrates both cases. The case in which there is ‘strict inflation-targeting rule’, when $\lambda = 0.0$, (in the left column), exhibits the following features.

1. The fluctuations in the optimal output path are slightly greater than those under the ‘flexible targeting rule’, when $\lambda = 0.5$. On the other hand, the optimal inflation path is stable, at around $\pi^* = 1.64$.

2. Similarly to when $\lambda = 0.5$, monetary tightening in the late 1980s was delayed.

3. Similarly to when $\lambda = 0.5$, monetary tightening in the early 1990s might have been excessive.

4. Contrary to when $\lambda = 0.5$, the optimal policy path from the early to the mid 1990s is identical to the actual policy path. This implies that this period’s moderate easing was reasonable given the rule.

5. If the BOJ had adopted the strict inflation-targeting rule of $\pi^* = 1.64$ from the late 1990s, it could have overcome deflation by substantially
easing policy. However, policy easing would not have necessarily raised output.

The case of the ‘strict output-targeting rule’, when $\lambda = 1.0$ (in the right column), exhibits the following features.

1. The fluctuations in the optimal output path are much smaller than those under the ‘flexible targeting rule’ and the ‘strict inflation targeting rule’, whereas the fluctuations in the optimal inflation path are much larger.

2. Similarly to when $\lambda = 0.5$ and $\lambda = 0.0$, the optimal policy path suggests delayed tightening in the late 1980s.

3. Similarly to when $\lambda = 0.5$, and contrary to when $\lambda = 0.0$, monetary easing from the early to the mid 1990s was delayed.

4. Similarly to when $\lambda = 0.5$ and $\lambda = 0.0$, the optimal policy path in the early 1990s indicates that the substantial tightening that occurred in that period might have been excessive. However, when $\lambda = 1.0$, the optimal path in this period is closer to the actual path than when $\lambda = 0.5$ and $\lambda = 0.0$.

5. Contrary to when $\lambda = 0.5$ and $\lambda = 0.0$, the optimal policy path from the late 1990s fluctuates widely. In particular, the steep decline of the optimal path in August 2000 implies the temporary tightening involving raising the call rate was reasonable given the rule.

4 Conclusion

This paper has drawn three main conclusions.

First, we suggest that the call rate should be used as the policy indicator of the BOJ to February 2001, and that an equally weighted average of the
call rate and reserves should be used as the BOJ’s policy indicator from March 2001. These suggestions are noteworthy because no simple monetary measure represents the BOJ’s past policy decisions over time.

Second, evaluation of the BOJ’s policy, especially from the early 1990s, depends on the type of optimal policy rule used as a yardstick. When the strict inflation-targeting rule is used as the yardstick, there is evidence that the moderate expansion between the early and mid 1990s was reasonable. However, under the assumption that the BOJ committed itself to output stabilization-oriented rules, including the flexible targeting rule and the strict output-targeting rule, a more aggressive and expansionary policy was needed. The same comments apply to policy evaluation from the late 1990s. When the strict output-targeting rule is used as the yardstick, the evidence suggests that the temporary tightening in August 2000 was reasonable. However, under the assumption that the BOJ paid attention to inflation stabilization, including the flexible targeting rule and the strict inflation-targeting rule, a persistent easining was needed. These comments suggest that one should not make sweeping judgments based on a particular policy rule about whether monetary policy undertaken at a particular time is reasonable.

Third, whatever optimal policy rules are adopted as yardsticks, the following is possible: (1) the BOJ’s policy stance in the late 1980s was too expansionary and tightening in this period was delayed; (2) the tightening in the early 1990s was excessive.

In this paper, by applying Bernanke and Mihov’s methodology, we have presented a useful policy measure of the BOJ that represents its past policy decisions over time. One could use this indicator to, for example, implement impulse response analysis or analyze the dynamic responses of monetary policy to different macroeconomic shocks. Using this measure to complete
these tasks is left to future research.  

Appendix A: Constructing MO

• Construction of MO:
  First, we apply X11 to foreign assets (net), claims on government, claims on deposit-money banks, lending to deposit-money banks and unclassified assets (net). Second, we subtract lending to deposit-money banks (SA) from the claims on deposit-money banks (SA). The transformed data measure claims that the BOJ acquires via open-market operations on deposit-money banks. Then, we define the sum of the transformed data, foreign assets (SA), claims on government (SA) and the unclassified assets (SA) as MO: the BOJ’s assets held via open-market operations. All data were obtained from Nikkei NEEDS (Monetary Survey, Accounts of Monetary Authority).

• Construction of MD:
  After applying X11 to lending to deposit-money banks, we define the sum of lending (SA) and MO as MD: the BOJ’s assets held via open-market operations and discount-window lending.

Appendix B: The Lucas Critique and Stability of the VAR

Optimal policy measures are derived under the assumption that the non-policy sector of the reduced-form VAR, in the form of equation (15), is invariant to policy rules chosen by the BOJ and is therefore subject to the Lucas (1976) critique. However, for the U.S. postwar period, Rudebusch (2005) has recently argued that reduced forms, such as VARs, of forward-looking macroeconomic specifications are insensitive to the policy-rule shifts

\footnotesize{\textsuperscript{25}The resulting policy measure is obtainable from the author. \\
\textsuperscript{26}SA denotes seasonally adjusted data.}
identified by Clarida et al. (2000), Estrella and Fuhrer (2003) and Taylor (1999); hence, the Lucas critique does not necessarily apply to estimated autoregressive reduced forms.  

In the context of Japanese monetary history, Jinushi et al. (2000) have identified a policy-rule shift in the response of the BOJ to inflation and output over the past twenty years. Given the observation of Jinushi et al., the stability of the nonpolicy sector in the reduced-form VAR is required if Rudebusch’s finding is to apply to our data set. Hence, in this appendix, we statistically examine the stability of the nonpolicy sector in the reduced-form VAR of equation (15).

We employ three types of Lagrange multiplier (LM) test, the Sup LM, the Exp LM, and the Ave LM test, proposed by Andrews (1993) and Andrews and Ploberger (1994). For the tests of structural change, the null hypotheses of the parameter stability of the output gap and that of the inflation equation are tested against the corresponding alternative hypotheses of parameter instability. Further, we use the methodology presented by Hansen (1997) to calculate asymptotic $p$-values for the structural-change tests.  

Table 6 reports the test results for parameter stability of the output gap and the inflation equation. For our estimated output-gap equation, the $p$-values of the Sup LM, the Exp LM and the Ave LM test statistics are 0.07, 0.08 and 0.17, respectively. For the inflation equation, the corresponding $p$-values are 0.14, 0.13 and 0.40. Hence, our estimated equations for the nonpolicy sector pass the structural-change tests.

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28 We compute the LM test statistics and the corresponding $p$-values by using the GAUSS code programmed by Professor Bruce Hansen. The LM statistics are computed by using the middle 70 percent of the sample.
References


### Table 1: The BOJ’s Balance Sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount-window Lending((u^{d}))</td>
<td>Government Deposits((u^{g}))</td>
</tr>
<tr>
<td>Assets Held via Open-Market Operations((u^{mo}))</td>
<td>Currency Held by the Public((u^{cu}))</td>
</tr>
<tr>
<td>(Security, Float, Other Net Assets)</td>
<td>Bank Deposits((u^{re}))</td>
</tr>
</tbody>
</table>

### Table 2: Alternative Models for the BOJ’s Operating Procedures

**CR Model (Before June 1995)**

<table>
<thead>
<tr>
<th>Models</th>
<th>BOJ Equations</th>
<th>Monetary Policy Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL (Call Rate)</td>
<td>(\psi^{gd}), (\psi^{cu}), (\psi^{re})</td>
<td>(v^{md} = -(\alpha + \beta)u^{r})</td>
</tr>
<tr>
<td>HP (High-powered Money)</td>
<td>(\psi^{gd}), (\psi^{cu}), (\psi^{re})</td>
<td>(v^{md} = u^{hp}) (= (u^{re} + u^{cu}))</td>
</tr>
<tr>
<td>CL-CU-RE</td>
<td>(\psi^{gd}), (\psi^{cu}), (\psi^{re})</td>
<td>(v^{md} = -(\alpha\psi^{cu} + \beta\psi^{re})u^{r} + (1 - \psi^{re})u^{ce} + (1 - \psi^{cu})u^{cu})</td>
</tr>
<tr>
<td>CL-CU</td>
<td>(\psi^{gd}), (\psi^{cu}), (\psi^{re})</td>
<td>(v^{md} = -(\alpha + \beta)u^{r} + (1 - \psi^{cu})u^{cu})</td>
</tr>
<tr>
<td>CL-RE</td>
<td>(\psi^{gd}), (\psi^{cu}), (\psi^{re})</td>
<td>(v^{md} = -(\alpha + \beta)u^{r} + (1 - \psi^{re})u^{re})</td>
</tr>
<tr>
<td>CL-HP</td>
<td>(\psi^{gd}), (\psi^{cu})</td>
<td>(v^{md} = -((\alpha + \beta)\psi^{cu})u^{r} + (1 - \psi^{cu})u^{hp})</td>
</tr>
</tbody>
</table>

**LIP Model (After July 1995)**

<table>
<thead>
<tr>
<th>Models</th>
<th>BOJ Equations</th>
<th>Monetary Policy Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL (Call Rate)</td>
<td>(\theta^{gd}), (\theta^{cu}), (\theta^{re}), -1.00</td>
<td>(v^{s} = -(\alpha + \beta)u^{r})</td>
</tr>
<tr>
<td>HP (High-powered Money)</td>
<td>(\theta^{gd}), (\theta^{cu}), (\theta^{re}), -1.00</td>
<td>(v^{s} = u^{hp}) (= (u^{re} + u^{cu}))</td>
</tr>
<tr>
<td>CL-CU-RE</td>
<td>(\theta^{gd}), (\theta^{cu}), (\theta^{re}), -1.00</td>
<td>(v^{s} = -(\alpha\theta^{cu} + \beta\theta^{re})u^{r} + (1 - \theta^{re})u^{re} + (1 - \theta^{cu})u^{cu})</td>
</tr>
<tr>
<td>CL-CU</td>
<td>(\theta^{gd}), (\theta^{cu}), (\theta^{re}), -1.00</td>
<td>(v^{s} = -(\alpha\theta^{cu} + \beta)u^{r} + (1 - \theta^{cu})u^{cu})</td>
</tr>
<tr>
<td>CL-RE</td>
<td>(\theta^{gd}), (\theta^{cu}), (\theta^{re}), -1.00</td>
<td>(v^{s} = -(\alpha + \beta\theta^{re})u^{r} + (1 - \theta^{re})u^{re})</td>
</tr>
<tr>
<td>CL-HP</td>
<td>(\theta^{gd}), (\theta^{cu}), -1.00</td>
<td>(v^{s} = -((\alpha + \beta)\theta^{cu} + \gamma)u^{r} + (1 - \theta^{cu})u^{hp})</td>
</tr>
</tbody>
</table>

1. CL, HP, CU and RE imply the call rate, high-powered money, currency and reserves, respectively.
2. We describe the CL and HP models as single-targeting models, which assume that the BOJ targets a single-monetary variable.
3. We describe the CL-CU-RE, CL-CU, CL-RE and CL-HP models as mixed-targeting models, which assume that the BOJ targets a combination of the following policy-sector variables: CL, HP, CU and RE.
Table 3: Estimation Results from the Credit Rationing (CR) Model (1976:1–1995:6)

<table>
<thead>
<tr>
<th>Models</th>
<th>Demand Equations</th>
<th>BOJ Equations</th>
<th>OIR</th>
<th>JOINT</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
<td>( \beta )</td>
<td>( \omega_{\text{CR}} )</td>
<td>( \omega_{\text{CL}} )</td>
<td>( \omega_{\text{HP}} )</td>
</tr>
<tr>
<td>CR</td>
<td>0.04 (0.05)</td>
<td>0.05 (0.06)</td>
<td>0.99 (0.01)</td>
<td>0.99 (0.12)</td>
<td>0.99 (0.01)</td>
</tr>
<tr>
<td>CL</td>
<td>0.02 (0.02)</td>
<td>0.02 (0.02)</td>
<td>1.00 (0.00)</td>
<td>1.00 (0.00)</td>
<td>1.00 (0.00)</td>
</tr>
<tr>
<td>HP</td>
<td>0.03 (0.01)</td>
<td>0.03 (0.00)</td>
<td>1.00 (0.00)</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>CL-CU-RE</td>
<td>0.07 (0.04)</td>
<td>0.07 (0.05)</td>
<td>1.00 (0.00)</td>
<td>0.99 (0.00)</td>
<td>0.99 (0.00)</td>
</tr>
<tr>
<td>CL-CU</td>
<td>0.06 (0.01)</td>
<td>0.02 (0.02)</td>
<td>1.00 (0.01)</td>
<td>1.00 (0.00)</td>
<td>1.00 (0.00)</td>
</tr>
<tr>
<td>CL-RE</td>
<td>0.01 (0.01)</td>
<td>0.02 (0.02)</td>
<td>1.00 (0.01)</td>
<td>1.00 (0.01)</td>
<td>0.99 (0.00)</td>
</tr>
<tr>
<td>CL-HP</td>
<td>0.07 (0.05)</td>
<td>0.07 (0.04)</td>
<td>1.00 (0.03)</td>
<td>0.99 (0.03)</td>
<td>0.99 (0.03)</td>
</tr>
</tbody>
</table>

1. For the Demand Equations and BOJ Equations, standard errors are in parentheses.
2. OIR and Joint indicate overidentifying restrictions test statistics and joint test statistics, respectively. p-values are in parentheses.
3. A likelihood ratio test was used to test the overidentifying restrictions. The degrees of freedom are one for the CR model, four for the CL and HP models, two for the CL-CU-RE model and three for the CL-CU, CL-RE and CL-HP models.
4. A likelihood ratio test was used to test the joint hypotheses. The degrees of freedom are three for the CL and HP models, one for the CL-CU-RE model and two for the CL-CU, CL-RE and CL-HP models.
Table 4: Estimation Results from the Low Interest Rates (LIP) Model (1995:7–2003:5)

### First Subperiod (1995:7–2001:2)

<table>
<thead>
<tr>
<th>Models</th>
<th>Demand Equations</th>
<th>BOJ Equations</th>
<th>OIR</th>
<th>JOINT</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$g_\text{pd}$</td>
<td>$g_\text{cu}$</td>
<td>$g_\text{re}$</td>
</tr>
<tr>
<td>LIP</td>
<td>0.01</td>
<td>0.23</td>
<td>0.99 (0.04)</td>
<td>0.91 (0.04)</td>
<td>0.91 (0.02)</td>
</tr>
<tr>
<td>CL</td>
<td>0.02</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>HP</td>
<td>0.04</td>
<td>0.53</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CL-CU-RE</td>
<td>0.07</td>
<td>0.02</td>
<td>1.00</td>
<td>0.99</td>
<td>0.95</td>
</tr>
<tr>
<td>CL-CU</td>
<td>0.09</td>
<td>0.00</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>CL-RE</td>
<td>0.05</td>
<td>0.10</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>CL-HP</td>
<td>0.03</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

1. For the Demand Equations and BOJ Equations, standard errors are in parentheses.
2. OIR and Joint indicate overidentifying restrictions test statistics and joint test statistics, respectively. p-values are in parentheses.
3. A likelihood ratio test was used to test the overidentifying restrictions. The degrees of freedom are four for the LIP model, eight for the CL and HP models, six for the CL-CU-RE model and seven for the CL-CU, CL-RE and CL-HP models.
4. A likelihood ratio test was used to test the joint hypotheses. The degrees of freedom are four for the CL and HP models, two for the CL-CU-RE model and three for the CL-CU, CL-RE and CL-HP models.

### Second Subperiod (2001:3–2003:5)

<table>
<thead>
<tr>
<th>Models</th>
<th>Demand Equations</th>
<th>BOJ Equations</th>
<th>OIR</th>
<th>JOINT</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$g_\text{pd}$</td>
<td>$g_\text{cu}$</td>
<td>$g_\text{re}$</td>
</tr>
<tr>
<td>LIP</td>
<td>0.08</td>
<td>0.12</td>
<td>0.97 (0.04)</td>
<td>0.75 (0.15)</td>
<td>0.32 (0.12)</td>
</tr>
<tr>
<td>CL</td>
<td>0.06</td>
<td>0.14</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>HP</td>
<td>0.14</td>
<td>0.86</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CL-CU-RE</td>
<td>0.06</td>
<td>0.42</td>
<td>1.00</td>
<td>0.91</td>
<td>0.58</td>
</tr>
<tr>
<td>CL-CU</td>
<td>0.15</td>
<td>0.15</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>CL-RE</td>
<td>0.05</td>
<td>0.42</td>
<td>1.00</td>
<td>1.00</td>
<td>0.59</td>
</tr>
<tr>
<td>CL-HP</td>
<td>0.05</td>
<td>0.02</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

1. For the Demand Equations and BOJ Equations, standard errors are in parentheses.
2. OIR and Joint indicate overidentifying restrictions test statistics and joint test statistics, respectively. p-values are in parentheses.
3. A likelihood ratio test was used to test the overidentifying restrictions. The degrees of freedom are four for the LIP model, eight for the CL and HP models, six for the CL-CU-RE model and seven for the CL-CU, CL-RE and CL-HP models.
4. A likelihood ratio test was used to test the joint hypotheses. The degrees of freedom are four for the CL and HP models, two for the CL-CU-RE model and three for the CL-CU, CL-RE and CL-HP models.
Table 5: Performance of Policy Rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>Std. dev. (y)</th>
<th>Std. dev. (π)</th>
<th>Loss. fn.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Policy</td>
<td>3.34</td>
<td>1.96</td>
<td>-11.9</td>
</tr>
<tr>
<td>A (λ = 1.00)</td>
<td>1.17</td>
<td>3.13</td>
<td>-1.17</td>
</tr>
<tr>
<td>B (λ = 0.75)</td>
<td>1.37</td>
<td>2.53</td>
<td>-91.2</td>
</tr>
<tr>
<td>C (λ = 0.54)</td>
<td>1.82</td>
<td>1.96</td>
<td>-136</td>
</tr>
<tr>
<td>D (λ = 0.50)</td>
<td>1.92</td>
<td>1.86</td>
<td>-141</td>
</tr>
<tr>
<td>E (λ = 0.25)</td>
<td>2.76</td>
<td>1.07</td>
<td>-140</td>
</tr>
<tr>
<td>F (λ = 0.12)</td>
<td>3.34</td>
<td>0.60</td>
<td>-141</td>
</tr>
<tr>
<td>G (λ = 0.00)</td>
<td>4.01</td>
<td>0.25</td>
<td>-40.7</td>
</tr>
</tbody>
</table>

1. A, D and F denote strict output targeting, flexible targeting and strict inflation targeting, respectively.
2. The optimal values of the loss function (16) are calculated by using $V(X) = \text{trace}(\Lambda \Sigma)$.
3. The standard deviations are calculated by using the doubling algorithm described in Anderson et al. (1996).

Table 6: Structural Change Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Output Gap</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test statistics</td>
<td>Asymptotic p-values</td>
</tr>
<tr>
<td>Sup LM</td>
<td>63.9</td>
<td>0.07</td>
</tr>
<tr>
<td>Exp LM</td>
<td>27.6</td>
<td>0.08</td>
</tr>
<tr>
<td>Ave LM</td>
<td>42.1</td>
<td>0.17</td>
</tr>
</tbody>
</table>

1. The output-gap equation and the inflation equation are both estimated with 12 lags.
2. LM denotes the Lagrange multiplier statistic of the null hypothesis of no structural change.
4. Asymptotic p-values for the structural-change tests are computed using the methodology proposed by Hansen (1997).
Figure 1: Call Rate and Discount Rate

Figure 2: The Bank of Japan’s Policy Measure
Figure 3: Optimal Policy Frontier
Figure 4: Actual and Optimal Paths when $\lambda = 0.50$
Figure 5: Actual and Optimal Paths when $\lambda = 0.00$ and $\lambda = 1.00$