

The Interaction Effect in a Nonlinear Specification of Bank Lending: A Replication and Reexamination of the Peek and Rosengren (2005) Results on “Unnatural Selection” *

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ABSTRACT. Peek and Rosengren (American Economic Review 2005; 95) suggested the mechanism of “unnatural selection,” where low-capitalized Japanese banks increase credit to low-quality firms because of their motivation to pursue balance sheet cosmetics. In this study, we replicate their estimation results, using loan-level data from 1994 to 1999, and thereby reexamine this mechanism in terms of the interaction effect in a nonlinear specification of bank lending. We demonstrate that their results imply that Japanese banks allocated lending from viable firms to unviable ones regardless of the degree of bank capitalization, being not consistent with the balance sheet cosmetics hypothesis.

JEL classification: G01, G21, G28.

Keywords: interaction effect; nonlinear specification; probit model; forbearance lending

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1. Introduction When borrowers become insolvent, a bank may become financially distressed. Such a financially distressed bank has incentives to continue to lend to insolvent borrowers to conceal its predicament, while hoping that the circumstance of insolvent borrowers will improve. This type of bank lending that hopes for a revival in the credit status of borrowers is called forbearance lending, evergreening lending, or zombie lending. If many banks engage in this type of lending, the resulting misallocation of credit to unviable firms that could go bankrupt would damage the macroeconomic situation further still (Hoshi (2006) and Caballero et al. (2008)). Hence, this practice has been considered to be the source of the prolonged economic stagnation experienced since the 1990s in Japan.

The empirical study by Peek and Rosengren (2005) is the most important and influential piece of research on the misallocation of bank credit in Japan.¹ They specified forbearance lending in a nonlinear function and used loan-level data from 1994 to 1999. They found that low-capitalized Japanese banks' attempts to avoid the realization of losses on their balance sheets (so-called balance sheet cosmetics herein) induces the mechanism of "unnatural selection," in which low-capitalized Japanese banks are more likely to provide additional credit to unviable firms.

In this study, we replicate their estimation results, thereby reassessing this mechanism in terms of the interaction effect in a nonlinear specification of bank lending. Thus, we rigorously demonstrate that Peek and Rosengren's (2005) estimation results imply that Japanese banks allocated lending from viable firms to unviable ones regardless of the degree of bank capitalization, being not consistent with the balance sheet cosmetics hypothesis.

The rest of the paper is as follows: Section 2 discusses the potential shortcoming of Peek and Rosengren's (2005) nonlinear specification of bank lending in terms of the interaction

¹ Empirical research on forbearance lending in Japan also includes Sekine et al. (2003) and Watanabe (2010). Sekine et al. (2003) used firm-level panel data from 1986 to 1999 and found that highly indebted firms belonging to nonmanufacturing industries are more likely to increase their bank borrowings for the sample period after 1993 despite their low level of profitability. Watanabe (2010) used bank-level panel data from 1995 to 2000, demonstrating that banks with large capital losses, particularly caused by the regulator's request in 1997 for the rigorous assessment of outstanding bank loans, are more likely to reallocate lending to unhealthy industries with a higher concentration of nonperforming loans. In terms of a theoretical framework, Bruche and Llobet (2014) provided a precondition for avoiding forbearance lending to low-quality firms. They suggested that regulators should induce banks to disclose the deterioration of their capital condition.

effect, Section 3 reexamines the implication of their estimation results, and the final section provides our conclusions.

2. Specifying Lending to Low-quality Firms by using a Nonlinear Function

Peek and Rosengren (2005) specified forbearance lending by using a random effects probit model with an interaction term consisting of a low-capitalized bank indicator and a firm performance variable (i.e., return on assets or working capital ratio). They set random effects terms for each firm as firm unobserved components. This section briefly explains the potential shortcoming of specifying the misallocation of bank credit in such a random effects probit model with an interaction term.²

To illustrate the essence of this econometric problem, following Ai and Norton (2003), we use a general functional form $F(\cdot)$ and write the conditional expected values of y as a function of the linear index function $v = \beta_1 x_1 + \beta_2 x_2 + \beta_{12}(x_1 \times x_2) + X_3 B_3$:

$$E[y|x_1, x_2, X_3] = F(v) = F(\beta_1 x_1 + \beta_2 x_2 + \beta_{12}(x_1 \times x_2) + X_3 B_3). \quad (1)$$

Function F could be the logit or probit transformation, the logarithmic or exponential transformation, or any other nonlinear function of the linear index function v . x_1 and x_2 denote the continuous variables to be used to construct the interaction term $x_1 \times x_2$ in the nonlinear function F . X_3 denotes a vector variable including other observable control variables.

In nonlinear equation (1), we can express the marginal effect of x_k ($k = 1$ or 2) on the conditional expected value of y as follows:³

$$\frac{\partial E[y|x_1, x_2, X_3]}{\partial x_k} = \frac{dF}{dv} \frac{dv}{dx_k} = (\beta_k + \beta_{12} x_l) \frac{dF}{dv} \quad \text{for } k \neq l. \quad (2)$$

Then, we can write the cross-partial derivative, or the so-called “interaction effect” in the

² The same problem of the statistical inference of an interaction term in a nonlinear equation has also been discussed in political science (e.g., Berry et al. (2010)).

³ To clarify the issue of the nonlinear specifications of bank lending, we assume that x_1 and x_2 are continuous variables. We discuss below the case where one of these two variables is an indicator variable, which is the case where Peek and Rosengren (2005) specified the forbearance lending with by using the a probit specification.

following equation:

$$\begin{aligned} \frac{\partial^2 E[y|x_1, x_2, X_3]}{\partial x_1 \partial x_2} &= \frac{\partial}{\partial x_1} \left[\frac{dF}{dv}(\beta_2 + \beta_{12}x_1) \right] = \frac{\partial}{\partial x_2} \left[\frac{dF}{dv}(\beta_1 + \beta_{12}x_2) \right] \\ &= \left[\frac{dF}{dv} \beta_{12} \right] + \left[\frac{d^2 F}{dv^2}(\beta_1 + \beta_{12}x_2)(\beta_2 + \beta_{12}x_1) \right]. \end{aligned} \quad (3)$$

Note that even if the coefficient of the interaction term, β_{12} , is zero, the expression above for the interaction effect, $\partial^2 E[y|x_1, x_2, X_3] / \partial x_1 \partial x_2$, still has a nonzero value. This means that the statistical significance of the interaction effect cannot be tested with that of the estimated coefficient of β_{12} . Further, the sign of β_{12} does not necessarily indicate that of the interaction effect.

In Peek and Rosengren (2005), y corresponds to an indicator variable, $\text{LOAN}_{i,j,t}$, which has a value of one if loans to firm i by bank j increases from year $t - 1$ to year t and zero otherwise. Thus, they performed the probit transformation of the linear index v in function $E[y|x_1, x_2, X_3] = \Pr(y = 1|x_1, x_2, X_3) = \Phi(v)$, where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Variable x_1 corresponds to a low-capitalized bank indicator $\text{REQ2}_{j,t-1}$, which is a (0,1) dummy variable that has a value of one if the bank's reported risk-based capital ratio is less than two percentage points above the bank's required capital ratio at year $t - 1$ and zero otherwise. Variable x_2 corresponds to the lagged variable of firm i 's financial capability, measured by using its profitability measured as return on assets $\text{FROA}_{i,t-1}$ or the working capital ratio $\text{FWORKCAP}_{i,t-1}$. Vector variable X_3 includes other lender-side and borrower-side observables. In their specification of bank lending, if the interaction effects of $\text{REQ2}_{j,t-1} \times \text{FROA}_{i,t-1}$ and $\text{REQ2}_{j,t-1} \times \text{FWORKCAP}_{i,t-1}$ have negative values, it implies that low-capitalized banks provide more credit to unviable firms (i.e., those that have lower profitability and lower financial health) than non-low-capitalized banks do.

A potential shortcoming of the approach of Peek and Rosengren (2005) is that their analysis with the probit estimation of the bank lending equation was based on the estimated coefficients of the bank financial health variable as well as the firm performance variables and their interaction terms, but not on the marginal effects and interaction effects, as expressed in equations (2) and (3). Their estimated coefficients of the bank financial health

indicator, $REQ2_{j,t-1}$, had significantly positive values, while those of the firm variables, $FROA_{i,t-1}$ and $FWORKCAP_{i,t-1}$, had significantly negative ones.

More importantly, they showed that the coefficients of the interaction terms, β_{12} , were estimated to be significantly negative, thereby arguing that low-capitalized banks were more likely to lend credit to low-quality firms; in other words, forbearance lending by low-capitalized banks to low-quality firms prevailed during the late 1990s in Japan. However, as discussed above, the negative values of the coefficients of the interaction terms do not necessarily ensure the existence of forbearance lending in terms of the interaction effects, $\partial^2 E[y|x_1, x_2, X_3]/\partial x_1 \partial x_2$. Rather, negative estimates of the interaction effects are necessary for the existence of forbearance lending to low-quality firms by low-capitalized banks, who have window-dressing motives to avoid the realization of losses on their balance sheets. In the next section, we thus reexamine this mechanism of unnatural selection by reporting the estimated interaction effects obtained with the same probit specification as that in Peek and Rosengren (2005).

3. Reexamination of Unnatural Selection In the random effects probit specification proposed by Peek and Rosengren (2005), the interaction effect in equation (3) is expressed as follows:

$$\begin{aligned} \frac{\Delta \partial E[y|x_1, x_2, X_3, r_i]}{\Delta x_1 \partial x_2} &= \frac{\Delta \partial \Pr[y = 1|x_1, x_2, X_3, r_i]}{\Delta x_1 \partial x_2} = \frac{\partial \Phi(v + r_i)}{\partial x_2} \Big|_{x_1=1} - \frac{\partial \Phi(v + r_i)}{\partial x_2} \Big|_{x_1=0} \\ &= \phi(\beta_1 + \beta_2 x_2 + \beta_{12} x_2 + X_3 B_3 + r_i)(\beta_2 + \beta_{12}) \\ &\quad - \phi(\beta_2 x_2 + X_3 B_3 + r_i)\beta_2, \end{aligned} \quad (4)$$

where $\phi(\cdot) = \Phi'(\cdot)$ is the probability density function of the standard normal distribution. x_1 is the low-capitalized bank indicator, $REQ2_{j,t-1}$, and x_2 is firm performance measured by using the return on assets, $FROA_{i,t-1}$ or working capital ratio, $FWORKCAP_{i,t-1}$. r_i indicates firm i 's random effects.⁴

⁴ To estimate the interaction effect using the random effects probit specification, we must obtain a marginal prediction with respect to the firm random effects, $\Pr(y = 1|x_1, x_2, X_3)$. We compute the marginal prediction by integrating a conditional prediction, $\Pr(y = 1|x_1, x_2, X_3, r_i)$, with respect to the firm random effects over their support; more specifically, $\Pr(y = 1|x_1, x_2, X_3) = \int \Phi(v + r_i)g(r_i|\sigma^2)dr_i$, where $g(r_i|\sigma^2)$ indicates the $N(0, \sigma^2)$ density function of the random effects.

Equation (4) clarifies the essence of the empirical analysis based on the interaction terms of the low-capitalized bank indicator and firm performance variables. The first term in this equation represents the marginal effect of bank j 's loans to firm i with respect to firm i 's profitability ($x_2 = \text{FROA}_{i,t-1}$ or $\text{FWORKCAP}_{i,t-1}$) in the case where bank j is low-capitalized (i.e., $x_1 = \text{REQ2}_{j,t-1} = 1$). The second term indicates the marginal effect in the case where the bank is not low-capitalized (i.e., $x_1 = \text{REQ2}_{j,t-1} = 0$). This decomposition of the interaction effect allows us to rigorously analyze the lending behavior of all banks in Japan regardless of their degree of bank capitalization.⁵

Let us express the former marginal effect evaluated at a hypothetical value of firm performance, $x_2 = \dot{x}_2$, as $\text{ME}_{i,j,t}(\text{REQ2}_{j,t-1} = 1, x_2 = \dot{x}_2)$ and the latter marginal effect as $\text{ME}_{i,j,t}(\text{REQ2}_{j,t-1} = 0, x_2 = \dot{x}_2)$. Then, we obtain a consistent estimator for the average interaction effect evaluated at a hypothetical value of firm performance (hereafter, $\text{AIE}(x_2 = \dot{x}_2)$) as the sample mean of the interaction effect (4):

$$\begin{aligned}
\text{AIE}(x_2 = \dot{x}_2) &= \frac{1}{n} \sum_{i,j,t} \left[\left. \frac{\partial \Phi(v + r_i)}{\partial x_2} \right|_{x_1=1, x_2=\dot{x}_2} \right] - \frac{1}{n} \sum_{i,j,t} \left[\left. \frac{\partial \Phi(v + r_i)}{\partial x_2} \right|_{x_1=0, x_2=\dot{x}_2} \right] \\
&= \frac{1}{n} \sum_{i,j,t} \text{ME}_{i,j,t}(\text{REQ2}_{j,t-1} = 1, x_2 = \dot{x}_2) \\
&\quad - \frac{1}{n} \sum_{i,j,t} \text{ME}_{i,j,t}(\text{REQ2}_{j,t-1} = 0, x_2 = \dot{x}_2) \\
&= \text{AME}(\text{REQ2}_{j,t-1} = 1, x_2 = \dot{x}_2) \\
&\quad - \text{AME}(\text{REQ2}_{j,t-1} = 0, x_2 = \dot{x}_2), \tag{5}
\end{aligned}$$

where n denotes the number of observations (bank–firm relationships).⁶ We calculate the

⁵ Once we introduce the interaction terms, we cannot derive correct inferences about the lending behavior prevailing in the Japanese banking system without comparing the lending behavior of non-low-capitalized and low-capitalized banks, as shown in equations (4) and (5). Peek and Rosengren (2005) obtained negative estimates for a coefficient parameter of firm profitability, thus suggesting that Japanese banks lend more credit to low-quality firms through evergreening lending. However, their interpretation of these negative estimates is unsuitable for their empirical analysis based on the interaction terms.

⁶ More strictly, the first and second terms in equation (5) respectively represent the counterfactual effects in the hypothetical cases where all banks are low-capitalized and non-low-capitalized ones; hence, the interaction effect measures the treatment effect of the bank's low capitalization, expressed in $\text{REQ2}_{j,t-1} = 1$, as long as the confounding factors that can affect bank capitalization are fully controlled for in the bank lending equation by using X_3 .

standard errors of the average marginal effects by using the delta method (for the details of the calculation, see online Appendices A and B).

To reexamine the implication of Peek and Rosengren’s (2005) estimation results with the average interaction effects, we start by replicating the estimation results from their dataset of AER Final Data.txt, which is available online at the journal website. The analysis of the average marginal effects presented below is based on the replication of the “Full sample” results reported in Table 5 of Peek and Rosengren (2005). The sample period runs from 1994 to 1999, during which Japanese banks faced increasing pressure to maintain regulatory capital requirements under the Basel I framework.

Table 1 reports the estimated coefficients of and descriptive statistics for the bank’s financial health indicator, $x_1 = \text{REQ2}_{j,t-1}$, and firm performance variables measured by using the return on assets, $x_2 = \text{FROA}_{i,t-1}$, and working capital ratio, $x_2 = \text{FWORKCAP}_{i,t-1}$.⁷ The descriptive statistics for $\text{REQ2}_{j,t-1}$ indicate that about 70 percent of Japanese banks showed a low degree of capitalization in the late 1990s. The estimation results in this table clearly show that we can replicate Peek and Rosengren’s (2005) estimation results: $\text{REQ2}_{j,t-1}$ has a significantly positive coefficient, while $\text{FROA}_{i,t-1}$ and $\text{FWORKCAP}_{i,t-1}$ have significantly negative ones. The interaction terms, $\text{REQ2}_{j,t-1} \times \text{FROA}_{i,t-1}$ and $\text{REQ2}_{j,t-1} \times \text{FWORKCAP}_{i,t-1}$, appear to have significantly negative coefficients.⁸

Figure 1 reports the estimated interaction effects and marginal effects obtained by using the firm performance variables, $\text{FROA}_{i,t-1}$ (upper panels) and $\text{FWORKCAP}_{i,t-1}$ (lower panels). The left-hand side panels show the estimated interaction effects, $\text{AIE}(x_2 = \hat{x}_2)$, while the right-hand side panels show the estimated marginal effects for low- and non-low-capitalized banks, $\text{AME}(\text{REQ2}_{j,t-1} = 1, x_2 = \hat{x}_2)$ and $\text{AME}(\text{REQ2}_{j,t-1} = 0, x_2 = \hat{x}_2)$. The interaction effect and marginal effect are estimated at each hypothetical value of firm performance, $x_2 = \hat{x}_2$, whose range corresponds to the sample range from -25 to 25 for

⁷ Peek and Rosengren (2005) defined the firm’s return on assets as $\text{FROA}_{i,t} = \text{Operating Profit}_{i,t} / \text{Total Asset}_{i,t-1} \times 100$ and the firm’s working capital ratio as $\text{FWORKCAP}_{i,t} = (\text{Liquid Asset}_{i,t} - \text{Current Liability}_{i,t}) / \text{Total Asset}_{i,t-1} \times 100$.

⁸ Our random effects probit regression includes all the other control variables and reproduces the same estimation results for these as in Peek and Rosengren’s (2005) regression, though Table 1 does not report the results. For the estimation results of the other control variables, see Table 5 in Peek and Rosengren (2005).

return on assets and from -120 to 80 for the working capital ratio.

The shaded areas of each panel also report the histograms of the two firm variables to allow us to analyze the allocation of bank credit in association with the actual performance of borrowing firms in the late 1990s. The histograms illustrate that few Japanese firms borrowing capital in the late 1990s suffered from low profitability and/or low financial capability. Indeed, nine-tenths and three-quarters of the distribution of the return on assets and working capital ratio show positive values, respectively.

Note that the estimated interaction effects, $AIE(x_2 = \hat{x}_2)$, in the left-hand side panels have significantly negative values for most of the hypothetical values of the return on assets and firm working capital ratio. This finding indicates not only that low-capitalized banks were more likely to increase loans to unviable firms than were non-low-capitalized banks in the study period, as suggested by Peek and Rosengren (2005), but also that low-capitalized banks were more likely to decrease loans to viable firms than non-low-capitalized banks were (or leave them unchanged). Rather, given that most firms borrowing capital performed well in the late 1990s, the negative values of the interaction effects imply that the misallocation of credit from viable firms to unviable ones prevailed because of low-capitalized banks' motivation to pursue balance sheet cosmetics.

More importantly, the right-hand side panels in Figure 1 show that the marginal effects for low- and non-low-capitalized banks, $AME(REQ2_{j,t-1} = 1, x_2 = \hat{x}_2)$ and $AME(REQ2_{j,t-1} = 0, x_2 = \hat{x}_2)$, have significantly negative estimates.⁹ This finding clearly indicates that the misallocation of bank credit from viable firms to unviable ones prevailed in the Japanese banking sector in the late 1990s; in other words, Japanese banks provided more credit to relatively unviable firms, while decreasing credit to viable ones (or keeping it unchanged) regardless of the degree of bank capitalization. This lending behavior by capitalized banks is not consistent with the balance sheet cosmetics hypothesis.

4. Conclusions The mechanism of unnatural selection suggested by Peek and Rosengren (2005) assumes that forbearance lending by low-capitalized banks to low-quality bor-

⁹ Although we defined the low-capitalized bank indicator, $REQ2_{j,t-1}$, by setting the threshold value of bank capital buffers above the minimum requirement to various values less than two percentage points, we confirm that the average marginal effects for both low- and non-low-capitalized banks have significantly negative estimates.

rowers prevailed during the late 1990s in Japan, particularly driven by banks' motivation to pursue balance sheet cosmetics. In this study, we reevaluated this mechanism by replicating their estimation results and then focusing on the interaction effect instead of the coefficient parameter of the interaction term. More concretely, we discussed a potential shortcoming of specifying bank lending by using nonlinear functions, demonstrating that their estimation results, which are based on the random effects probit model, imply that Japanese banks allocated lending from viable firms to unviable ones in the late 1990s regardless of the degree of bank capitalization, although low-capitalized banks were still more likely to do so than non-low-capitalized banks. Their estimation results compel us to explore other hypotheses besides the balance sheet cosmetics one to explain why Japan's banking system emphasized relationships with relatively low-quality firms.

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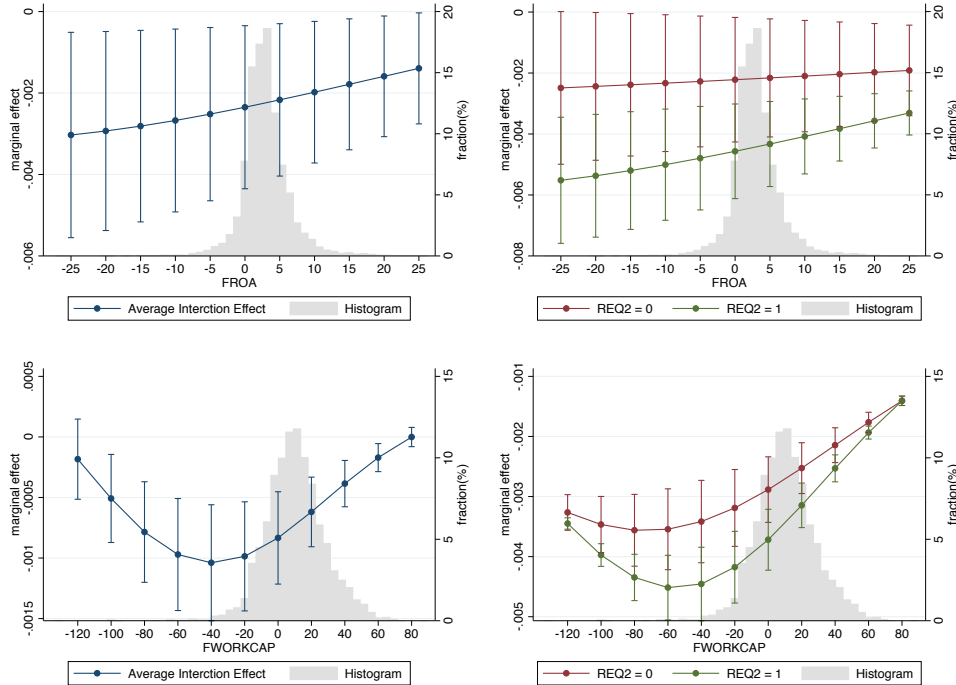
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Table 1: Estimated Coefficients of the Bank Financial Health Variable, Firm Performance Variable, and Their Interaction Term (1994 – 1999)

	Estimated Coefficients (Standard Errors)		Descriptive Statistics		
	Our Estimation	Peek and Rosengren	Mean (Std. Dev.)	Min.	Max.
REQ2	0.0556** (0.0164)	0.0582** (0.0157)	0.714 (0.452)	0	1
FROA	-0.0086* (0.0040)	-0.0075** (0.0028)	2.970 (3.292)	-27.291	26.193
FWORKCAP	-0.0107** (0.0010)	-0.0097** (0.0006)	10.283 (17.431)	-133.592	89.354
REQ2 × FROA	-0.0088* (0.0039)	-0.0095* (0.0034)			
REQ2 × FWORKCAP	-0.0029** (0.0007)	-0.0030** (0.0006)			
Number of Observations		95566			
Number of Firms		1215			

Notes: The estimation results are obtained from the random effects probit regression. The regression also includes all the other control variables of the full sample model reported in Table 5 of Peek and Rosengren (2005). Peek and Rosengren’s (2005) results are obtained from their Table 5. For the estimation results, standard errors are in parentheses. * Significant at the 5 percent level; ** significant at the 1 percent level.

Figure 1: Average Interaction Effects and Average Marginal Effects



Notes: The dots (left axis) indicate the estimated average effects and the capped spikes indicate their 95% confidence intervals. The shaded areas (right axis) report a histogram of the firm performance variable.

Online Appendix

“The Interaction Effect in a Nonlinear Specification of Bank Lending: A Replication and Reexamination of the Peek and Rosengren (2005) Results on “Unnatural Selection”” by Inoue, Nakashima, and Takahashi

In Appendix A, we explain how to calculate the average marginal effects and their standard errors for the probit model with an interaction term consisting of an indicator variable and a continuous variable. In Appendix B, we consider the random effects probit model with an interaction term.

Appendix A. Probit Model with an Interaction Term

First, we consider a probit model with an interaction term as follows:

$$\begin{aligned}\Pr(y = 1|x_1, x_2, X_3) &= \Phi(v), \\ v &= \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + X_3 B_3,\end{aligned}\tag{A-1}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and $x_1 x_2$ is an interaction term of an indicator variable x_1 and a continuous variable x_2 . In our lending equation specified in the main text, x_1 is a dummy variable that indicates whether an individual bank’s capital buffers are less than two percentage points, $REQ2_{j,t-1}$. x_2 is a firm performance variable measured by using the working capital ratio, $FWORKCAP_{i,t-1}$, or return on assets, $FROA_{i,t-1}$. X_3 denotes a $(1 \times k)$ vector including the other control variables. y corresponds to $LOAN_{i,j,t}$. β_1 , β_2 , β_{12} , and B_3 are the coefficient parameters.

A-I. Average Marginal Effects with respect to an Indicator Variable

We start by calculating the marginal effect of $\Pr(y = 1|x_1, x_2, X_3)$ with respect to the indicator variable x_1 as expressed in the following:

$$ME_1 = \frac{\Delta \Pr(y = 1|x_1, x_2, X_3)}{\Delta x_1} = \Pr(y = 1|x_1 = 1) - \Pr(y = 1|x_1 = 0).\tag{A-2}$$

Note that $\widehat{\text{ME}}_1$ is a random variable because the coefficient estimators $\hat{\beta}$'s and $\widehat{\text{B}}_3$ are random variables. A consistent estimator for the marginal effect is calculated as

$$\widehat{\text{ME}}_1 = \Phi(\hat{v}_1) - \Phi(\hat{v}_0), \quad (\text{A-3})$$

where

$$\begin{aligned} \hat{v}_1 &= \hat{\beta}_1 + \hat{\beta}_2 x_2 + \hat{\beta}_{12} x_2 + \text{X}_3 \widehat{\text{B}}_3, \\ \hat{v}_0 &= \hat{\beta}_2 x_2 + \text{X}_3 \widehat{\text{B}}_3. \end{aligned} \quad (\text{A-4})$$

The average marginal effect is calculated as a sample mean of the marginal effect $\widehat{\text{ME}}_1$, namely,

$$\begin{aligned} \widehat{\text{AME}}_1 &= \frac{1}{n} \sum^n [\Phi(\hat{v}_1) - \Phi(\hat{v}_0)] \\ &= \frac{1}{n} \sum^n [\Phi(\hat{v}_1)] - \frac{1}{n} \sum^n [\Phi(\hat{v}_0)], \end{aligned} \quad (\text{A-5})$$

where n denotes the number of observations.

We can compute the variance of the average marginal effect by using the delta method as follows:

$$\text{Var}(\widehat{\text{AME}}_1) = \mathbf{J}_1 \widehat{\mathbf{V}} \mathbf{J}_1', \quad (\text{A-6})$$

where $((3 + k) \times (3 + k))$ matrix $\widehat{\mathbf{V}}$ is the estimated variance-covariance matrix of the coefficient column vector $\widehat{\mathbf{b}} = [\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_{12}, \widehat{\text{B}}_3]'$ and a $(1 \times (3 + k))$ vector \mathbf{J}_1 is the Jacobian vector as expressed in the following derivatives:

$$\mathbf{J}_1 = \left[\frac{\partial \widehat{\text{AME}}_1}{\partial \mathbf{b}'} \Big|_{\mathbf{b}=\widehat{\mathbf{b}}} \right]. \quad (\text{A-7})$$

A (1, 1) element of the Jacobian vector \mathbf{J}_1 is

$$\begin{aligned} \left. \frac{\partial \text{AME}_1}{\partial \beta_1} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial \beta_1} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial \beta_1} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \{\phi(\hat{v}_1) \cdot 1\} - \frac{1}{n} \sum \{\phi(\hat{v}_0) \cdot 0\}; \end{aligned} \quad (\text{A-8})$$

a (1, 2) element is

$$\begin{aligned} \left. \frac{\partial \text{AME}_1}{\partial \beta_2} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial \beta_2} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial \beta_2} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \{\phi(\hat{v}_1) \cdot x_2\} - \frac{1}{n} \sum \{\phi(\hat{v}_0) \cdot x_2\}; \end{aligned} \quad (\text{A-9})$$

a (1, 3) element is

$$\begin{aligned} \left. \frac{\partial \text{AME}_1}{\partial \beta_{12}} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial \beta_{12}} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial \beta_{12}} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \{\phi(\hat{v}_1) \cdot x_2\} - \frac{1}{n} \sum \{\phi(\hat{v}_0) \cdot 0\}; \end{aligned} \quad (\text{A-10})$$

and the (1, 4) to (1, 3 + k) elements are

$$\begin{aligned} \left. \frac{\partial \text{AME}_1}{\partial B'_3} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial B'_3} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum \left\{ \left. \frac{\partial \Phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial B'_3} \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \{\phi(\hat{v}_1) \cdot X_3\} - \frac{1}{n} \sum \{\phi(\hat{v}_0) \cdot X_3\}, \end{aligned} \quad (\text{A-11})$$

where $\phi(\cdot) = \Phi'(\cdot)$ is the probability density function of the standard normal distribution.

A-II. Average Marginal Effects with respect to a Continuous Variable

Here, we calculate the marginal effect with respect to the continuous variable x_2 in the following:

$$\text{ME}_2 = \frac{\partial \Pr(y = 1|x_1, x_2, X_3)}{\partial x_2} = \frac{\partial \Phi(v)}{\partial v} \cdot \frac{\partial v}{\partial x_2} = \phi(v)(\beta_2 + \beta_{12}x_1). \quad (\text{A-12})$$

A consistent estimator for the average marginal effect is calculated as the sample mean of the marginal effect, $\widehat{\text{ME}}_2$, namely

$$\widehat{\text{AME}}_2 = \frac{1}{n} \sum \left[\phi(\hat{v})(\hat{\beta}_2 + \hat{\beta}_{12}x_1) \right]. \quad (\text{A-13})$$

The variance of the average marginal effect is constructed by using the delta method as follows:

$$\text{Var}(\widehat{\text{AME}}_2) = \mathbf{J}_2 \widehat{\mathbf{V}} \mathbf{J}_2'. \quad (\text{A-14})$$

In equation (A-14), the Jacobian vector \mathbf{J}_2 has the following elements: a (1, 1) element is

$$\begin{aligned} \left. \frac{\partial \text{AME}_2}{\partial \beta_1} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial}{\partial \beta_1} [\phi(v)(\beta_2 + \beta_{12}x_1)] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \left\{ \left. \frac{\partial \phi(v)}{\partial v} \frac{\partial v}{\partial \beta_1} (\beta_2 + \beta_{12}x_1) \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \left\{ -\hat{v} \cdot \phi(\hat{v})(\hat{\beta}_2 + \hat{\beta}_{12}x_1) \cdot x_1 \right\}; \end{aligned} \quad (\text{A-15})$$

a (1, 2) element is

$$\begin{aligned} \left. \frac{\partial \text{AME}_2}{\partial \beta_2} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial}{\partial \beta_2} [\phi(v)(\beta_2 + \beta_{12}x_1)] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \left\{ \left[\frac{\partial \phi(v)}{\partial v} \frac{\partial v}{\partial \beta_2} (\beta_2 + \beta_{12}x_1) + \phi(v) \right] \Big|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \left\{ -\hat{v} \cdot \phi(\hat{v})(\hat{\beta}_2 + \hat{\beta}_{12}x_1) \cdot x_2 \right\} + \frac{1}{n} \sum \left\{ \phi(\hat{v}) \right\}; \end{aligned} \quad (\text{A-16})$$

a (1, 3) element is

$$\begin{aligned} \left. \frac{\partial \text{AME}_2}{\partial \beta_{12}} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial}{\partial \beta_{12}} [\phi(v)(\beta_2 + \beta_{12}x_1)] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \left\{ \left[\frac{\partial \phi(v)}{\partial v} \frac{\partial v}{\partial \beta_{12}} (\beta_2 + \beta_{12}x_1) + \phi(v)x_1 \right] \Big|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum \left\{ -\hat{v} \cdot \phi(\hat{v})(\hat{\beta}_2 + \hat{\beta}_{12}x_1) \cdot x_1 x_2 \right\} + \frac{1}{n} \sum \left\{ \phi(\hat{v})x_1 \right\}; \end{aligned} \quad (\text{A-17})$$

and the (1, 4) to (1, 3 + k) elements are

$$\begin{aligned}
\left. \frac{\partial \text{AME}_2}{\partial B'_3} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum \left\{ \left. \frac{\partial}{\partial B'_3} [\phi(v)(\beta_2 + \beta_{12}x_1)] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\
&= \frac{1}{n} \sum \left\{ \left. \frac{\partial \phi(v)}{\partial v} \frac{\partial v}{\partial B'_3} (\beta_2 + \beta_{12}x_1) \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\
&= \frac{1}{n} \sum \left\{ -\hat{v} \cdot \phi(\hat{v})(\hat{\beta}_2 + \hat{\beta}_{12}x_1) \cdot X_3 \right\}.
\end{aligned} \tag{A-18}$$

A-III. Average Marginal Effects with respect to the Interaction Term

Finally, we calculate the average interaction effects, or the average marginal effect with respect to the interaction term x_1x_2 . To this end, we define the difference between the partial derivatives with respect to continuous variable x_2 evaluated at $x_1 = 1$ and $x_1 = 0$ as follows:

$$\begin{aligned}
\text{ME}_{12} &= \frac{\Delta \partial \text{Pr}(y = 1 | x_1, x_2, X_3)}{\Delta x_1 \partial x_2} = \left. \frac{\partial \Phi(v)}{\partial x_2} \right|_{x_1=1} - \left. \frac{\partial \Phi(v)}{\partial x_2} \right|_{x_1=0} \\
&= \phi(v)(\beta_2 + \beta_{12}x_1) \Big|_{x_1=1} - \phi(v)(\beta_2 + \beta_{12}x_1) \Big|_{x_1=0} \\
&= \phi(v_1)(\beta_2 + \beta_{12}) - \phi(v_0) \cdot \beta_2,
\end{aligned} \tag{A-19}$$

where

$$\begin{aligned}
v_1 &= \beta_1 + \beta_2x_2 + \beta_{12}x_2 + X_3B_3, \\
v_0 &= \beta_2x_2 + X_3B_3.
\end{aligned} \tag{A-20}$$

Then, the average marginal effect of the interaction term can be constructed as the sample mean of the marginal effect ME_{12} . We call this average marginal effect the average interaction effect AIE. The consistent estimator of the average interaction effect is

$$\begin{aligned}
\widehat{\text{AIE}} &= \frac{1}{n} \sum \left[\phi(\hat{v}_1)(\hat{\beta}_2 + \hat{\beta}_{12}) - \phi(\hat{v}_0) \cdot \hat{\beta}_2 \right] \\
&= \frac{1}{n} \sum \left[\phi(\hat{v}_1)(\hat{\beta}_2 + \hat{\beta}_{12}) \right] - \frac{1}{n} \sum \left[\phi(\hat{v}_0) \cdot \hat{\beta}_2 \right].
\end{aligned} \tag{A-21}$$

The variance of the average interaction effect is computed by using the delta method as follows:

$$\text{Var}(\widehat{\text{AIE}}) = \mathbf{J}_{12} \widehat{\mathbf{V}} \mathbf{J}'_{12}. \quad (\text{A-22})$$

The Jacobian vector \mathbf{J}_{12} has the following elements: a (1, 1) element is

$$\begin{aligned} \left. \frac{\partial \text{AIE}}{\partial \beta_1} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial \beta_1} [\phi(v_1)(\beta_2 + \beta_{12})] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial \beta_1} [\phi(v_0) \cdot \beta_2] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum^n \left\{ \left. \frac{\partial \phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial \beta_1} (\beta_2 + \beta_{12}) \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum^n \left\{ \left. \frac{\partial \phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial \beta_1} \cdot \beta_2 \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \quad (\text{A-23}) \\ &= \frac{1}{n} \sum^n \left\{ -\hat{v}_1 \cdot \phi(\hat{v}_1) (\hat{\beta}_2 + \hat{\beta}_{12}) \cdot 1 \right\} - \frac{1}{n} \sum^n \left\{ -\hat{v}_0 \cdot \phi(\hat{v}_0) \cdot \hat{\beta}_2 \cdot 0 \right\}; \end{aligned}$$

a (1, 2) element is

$$\begin{aligned} \left. \frac{\partial \text{AIE}}{\partial \beta_2} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial \beta_2} [\phi(v_1)(\beta_2 + \beta_{12})] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial \beta_2} [\phi(v_0) \cdot \beta_2] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum^n \left\{ \left[\left. \frac{\partial \phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial \beta_2} (\beta_2 + \beta_{12}) + \phi(v_1) \right] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &\quad - \frac{1}{n} \sum^n \left\{ \left[\left. \frac{\partial \phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial \beta_2} \cdot \beta_2 + \phi(v_0) \right] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\ &= \frac{1}{n} \sum^n \left\{ -\hat{v}_1 \cdot \phi(\hat{v}_1) (\hat{\beta}_2 + \hat{\beta}_{12}) \cdot x_2 \right\} + \frac{1}{n} \sum^n \left\{ \phi(\hat{v}_1) \right\} \\ &\quad - \frac{1}{n} \sum^n \left\{ -\hat{v}_0 \cdot \phi(\hat{v}_0) \cdot \hat{\beta}_2 \cdot x_2 \right\} - \frac{1}{n} \sum^n \left\{ \phi(\hat{v}_0) \right\}; \quad (\text{A-24}) \end{aligned}$$

a (1,3) element is

$$\begin{aligned}
\left. \frac{\partial \text{AIE}}{\partial \beta_{12}} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial \beta_{12}} [\phi(v_1)(\beta_2 + \beta_{12})] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial \beta_{12}} [\phi(v_0) \cdot \beta_2] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\
&= \frac{1}{n} \sum^n \left\{ \left[\left. \frac{\partial \phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial \beta_{12}} (\beta_2 + \beta_{12}) + \phi(v_1) \right] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\
&\quad - \frac{1}{n} \sum^n \left\{ \left[\left. \frac{\partial \phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial \beta_{12}} \cdot \beta_2 \right] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\
&= \frac{1}{n} \sum^n \left\{ -\hat{v}_1 \cdot \phi(\hat{v}_1)(\hat{\beta}_2 + \hat{\beta}_{12}) \cdot x_2 \right\} + \frac{1}{n} \sum^n \left\{ \phi(\hat{v}_1) \right\} \\
&\quad - \frac{1}{n} \sum^n \left\{ -\hat{v}_0 \cdot \phi(\hat{v}_0) \cdot \hat{\beta}_2 \cdot 0 \right\};
\end{aligned} \tag{A-25}$$

and the (1,4) to (1,3+k) elements are

$$\begin{aligned}
\left. \frac{\partial \text{AIE}}{\partial B'_3} \right|_{\mathbf{b}=\hat{\mathbf{b}}} &= \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial B'_3} [\phi(v_1)(\beta_2 + \beta_{12})] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum^n \left\{ \left. \frac{\partial}{\partial B'_3} [\phi(v_0) \cdot \beta_2] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\
&= \frac{1}{n} \sum^n \left\{ \left[\left. \frac{\partial \phi(v_1)}{\partial v_1} \frac{\partial v_1}{\partial B'_3} (\beta_2 + \beta_{12}) \right] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} - \frac{1}{n} \sum^n \left\{ \left[\left. \frac{\partial \phi(v_0)}{\partial v_0} \frac{\partial v_0}{\partial B'_3} \cdot \beta_2 \right] \right|_{\mathbf{b}=\hat{\mathbf{b}}} \right\} \\
&= \frac{1}{n} \sum^n \left\{ -\hat{v}_1 \cdot \phi(\hat{v}_1)(\hat{\beta}_2 + \hat{\beta}_{12}) \cdot X_3 \right\} - \frac{1}{n} \sum^n \left\{ -\hat{v}_0 \cdot \phi(\hat{v}_0) \cdot \hat{\beta}_2 \cdot X_3 \right\}.
\end{aligned} \tag{A-26}$$

A-IV. Average Interaction Effects at a Hypothetical Value

The average interaction effect can take various values because of the nonlinearity of the standard normal distribution function $\Phi(\cdot)$. Hence, we compute the average interaction effects evaluated at a hypothetical value of the continuous variable $x_2 = \hat{x}_2$ as follows:

$$\begin{aligned}
\text{AIE}(x_2 = \hat{x}_2) &= \frac{1}{n} \sum^n \left[\frac{\Delta \partial \Pr(y = 1 | x_1, x_2 = \hat{x}_2, X_3)}{\Delta x_1 \partial x_2} \right] \\
&= \frac{1}{n} \sum^n \left[\left. \frac{\partial \Phi(v)}{\partial x_2} \right|_{x_1=1, x_2=\hat{x}_2} \right] - \frac{1}{n} \sum^n \left[\left. \frac{\partial \Phi(v)}{\partial x_2} \right|_{x_1=0, x_2=\hat{x}_2} \right] \\
&= \frac{1}{n} \sum^n [\phi(\hat{v}_1)(\beta_2 + \beta_{12})] - \frac{1}{n} \sum^n [\phi(\hat{v}_0) \cdot \beta_2],
\end{aligned} \tag{A-27}$$

where

$$\begin{aligned} \dot{v}_1 &= \beta_1 + \beta_2 \dot{x}_2 + \beta_{12} \dot{x}_2 + X_3 B_3, \\ \dot{v}_0 &= \beta_2 \dot{x}_2 + X_3 B_3. \end{aligned} \tag{A-28}$$

Appendix B. Random Effects Probit Model with an Interaction Term

Next, we consider a random effects panel probit model with an interaction term as follows:

$$\begin{aligned} \Pr(y = 1 | x_1, x_2, X_3, r_i) &= \Phi(v + r_i), \\ v &= \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + X_3 B_3, \end{aligned} \tag{A-29}$$

where r_i denotes the firm random effects.

There are at least three ways of computing predictions when the model includes the firm random effects $r_i \sim N(0, \sigma^2)$ as follows:

1. $\Pr(y = 1 | x_1, x_2, X_3, r_i = 0) = \Phi(v)$, a prediction conditional on the random effects that are set to zeros.
2. $\Pr(y = 1 | x_1, x_2, X_3, r_i = \hat{r}_i) = \Phi(v + \hat{r}_i)$, a prediction conditional on the random effects that are estimated by the empirical Bayes means. The empirical Bayes means of the random effects, \hat{r}_i , are calculated as

$$\hat{r}_i = \frac{\int r_i \Phi(y | r_i, x_1, x_2, X_3; \hat{\beta}_1, \hat{\beta}_2, \hat{B}_3) \phi(r_i; \hat{\sigma}) dr_i}{\int \Phi(y | r_i) \phi(r_i) dr_i}, \tag{A-30}$$

where $\hat{\beta}_1, \hat{\beta}_2, \hat{B}_3$, and $\hat{\sigma}$ denote the estimated model parameters in equation (A-29).

The integration is conducted by using the adaptive Gaussian quadrature method.

3. $\Pr(y = 1 | x_1, x_2, X_3) = \int \Phi(v + r_i) g(r_i | \sigma^2) dr_i$, where $g(r_i | \sigma^2)$ is the $N(0, \sigma^2)$ density function. This is the unconditional prediction with respect to the random effects, which is calculated by integrating the conditional prediction with respect to the random effects over their support. The integration is conducted by using the adaptive

Gaussian quadrature method.